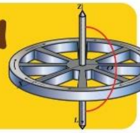
7.System of Particles and Rotational Motion

Physics Smart Booklet

Theory + NCERT MCQs + Topic Wise Practice MCQs + NEET PYQs

System of particles and rotational motion



1. Position of centre of mass depends upon shaped, size and distribution of mass of body
2. Position of centre of mass of an object changes in translation motion.
3. For bodies of normal dimensions centre of mass & center of gravity coincide.
4. Centre of mass of rigid bodies is independent of the state i.e rest or motion of the body.

CENTRE OF MASS

The point where whole mass of system is supposed to be concentrated

Position of Centre of mass of system

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

Velocity of centre of mass of system

$$\vec{V}_{cm} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$$

Acceleration of Centre of mass of system

$$\vec{a}_{cm} = \frac{\sum m_i \vec{a}_i}{\sum m_i}$$

RIGID BODY

A body with perfectly definite and unchanging shape.

Rotational Equilibrium

$$\vec{\tau}_{ext} = \varepsilon \vec{r} \times \vec{F}_{ext} = 0$$

Translational Equilibrium

$$\sum \vec{F}_{ext} = 0$$

PRINCIPLE OF MOMENTS FOR A LEVER

According to this principle:-

Load × Load arm = effort × effort arm

Factors & radius of gyration depends

(1) Position & configuration of the axis of rotation

(2) distribution of mass about the axis of Rotation.

MOMENT OF INERTIA

Inertia of Rotational motion

$$\text{M.I. } I = \sum_{i=1}^{n} m_i r_i^2 = MK^2$$

where r is distance perpendicular to the axis of Rotation.

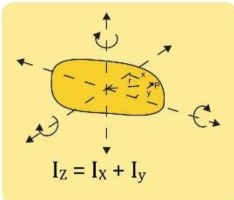
Radius of gyration

$$K = \sqrt{\frac{r_1^2 + r_2^2 \ldots\ldots\ldots r_n^2}{n}}$$

$$K = \sqrt{\frac{I}{M}}$$

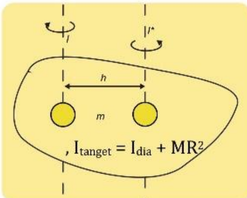
Theorem of moment of Inertia

Perpendicular axis theorem



$$I_Z = I_X + I_Y$$

Parallel – axis theorem



$$I_{tanget} = I_{dia} + MR^2$$

SHAPE OF AREA		DISTANCE X	DISTANCE Y	AREA
Square		a/2	a/2	a^2
Rectangle		a/2	b/2	ab
Circle		r	r	πr^2
Semi-circle		$4r/3\pi$	r	$\pi r^2/2$
Right-angled triangle		b/3	h/3	bh/2

ANGULAR MOMENTUM CONSERVATION

$\vec{L}_O = \vec{r}_{OA} \times \vec{P}$ (angular momentum about point O)

$= \vec{r}_{OA} \times (m\vec{v})$

$= m\vec{r}_{OA} \times \vec{v}$

$\vec{L}_O = \vec{r}_{OA} \times \vec{P} = r_{OA} P \sin\theta$

$= r_{OA}\, mv \sin\theta$

ANGULAR MOMENTUM CONSERVATION

$\vec{\tau}_{net} = \frac{d\tau}{dt}$ if $\vec{\tau}_{net} = 0 \Rightarrow \vec{L} = \text{constant}$

$$\vec{L}_{system} = \sum_{i=1}^{n} \vec{L}_i$$

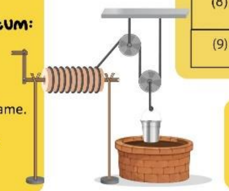
Angular momentum of rigid body performing pure rotation about fixed axis $(L_{sys})_{AOR} = I_{AOR}w$

Relation between Torque & Angular momentum:

- $\vec{\tau}_{net} = \frac{d\vec{t}}{dt}$
- Unit of Torque = N.m
- Dimensional formula = $[m^1 L^2 T^{-2}]$ Valid in only inertial frame.

Angular Impulse:- $\vec{J} = \int \vec{\tau}.dt$, $\vec{J}_{net} = \vec{L}_f - \vec{L}_i$, $\vec{J} = \vec{r} \times \vec{I}$, Unit → NmS

Linear Impulse:- $\vec{I} = \int \vec{F}\, dt$, $\vec{I}_{net} = \vec{P}_f - \vec{P}_i$, Unit → N.S



Analogy between linear & Rotational motion

Linear motion	Rotational Motion
Velocity $V = \frac{ds}{dt}$	Angular velocity $W = \frac{dQ}{dt}$
acceleration $a = \frac{ds}{dt}$	angular acceleration $\alpha = \frac{dw}{dt}$
Force $F = ma = \frac{mdv}{dt}$	torque $\tau = I\alpha = \frac{d}{dt}(Iw)$
work done $W = F.S$	work – done $W = \tau.Q$
linear K.E $\frac{1}{2}mv^2$	rotational K.E $\frac{1}{2}Iw^2$
Power $P = F.V$,	Power $P = \tau.W$,
Linear momentum $P = mv$	angular momentum $L = Iw$
Impulse $F\Delta t = mv - mu$	angular Impulse $\tau pt = Iw_f - Iw_i$

Z	Body	Axis	Figure	I
(1)	Thin circular ring, radius R	Perpendicular to plane, at centre	(figure)	MR^2
(2)	Thin circular ring, radius R	Diameter	(figure)	$MR^2/2$
(3)	Thin rod, lenght L	Perpendicular to rod, at mid point	(figure)	$ML^2/12$
(4)	Circular disc, radius R	Perpendicular to disc at centre	(figure)	$MR^2/2$
(5)	Circular disc, radius R	Diameter	(figure)	$MR^2/4$
(6)	Hollow cylinder, radius R	Axis of cylinder	(figure)	MR_2
(7)	Solid cylinder, radius R	Axis of cylinder	(figure)	$MR^2/2$
(8)	Solid cylinder, radius R	Diameter	(figure)	$2MR^2/5$
(9)	Hollow sphere, radius R	Diameter		$\frac{2}{3}mR^2$

(4) Time taken to reach the bottom of the inclined plane is.

$$t = \frac{1}{\sin\theta}\sqrt{\frac{2h\left(1+\frac{K^2}{R^2}\right)}{g}}$$

MOTION OF SYSTEM OF PARTICLES & RIGID BODY

Pure Rotational Motion:-

(1) Since distance between two particles of a rigid body remains constant. So the relative motion of one particle w.r.t other particle is circular motion.

(2) Angular velocity of all the particles about a given point of a Rigid body is same

$S = RQ,\ |V| = Rw$;

(3) If α = Constant (angular acceleration).

), $W_f = w_i + \alpha t$,

$Q_f = w_i t + \frac{1}{2}\alpha t^2\ w_f^2 =$

$w_i^2 + 2\alpha\theta,\ \theta = \left(\frac{w_i + w_f}{2}\right)t$

$\theta = w_f t - \frac{1}{2}\alpha t^2 \rightarrow K.E_{rolling} = \frac{1}{2}mv^2 + \frac{1}{2}Iw^2$,

$\frac{1}{2}mv^2 + \frac{1}{2}mk^2\left(\frac{V^2}{r^2}\right)$

$\frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$

Combined Rotation + translation Motion (CRTM):-

$\vec{V}_{CRTM} = \vec{V}_{pure\ rotation} + \vec{V}_{translational}$

$\vec{a}_{CRTM} = \vec{a}_{pure\ rotation} + \vec{a}_{translational}$

Dynamics of CRTM

for analysing its motion we apply two equation

$\sum \vec{\tau}_{ext} = M\vec{a}_{cm}$

$\sum \vec{\tau}_{ext} = I\vec{\alpha} = \vec{r} \times \vec{F}_{ext}$

Newton's laws of motion is valid in inertial frame.

To apply second equation of Newton about Non – inertial Point. Pseudo – force is applied at Com of body Σ of pseudo force is also taken into account.

$\rightarrow K.E_{CRTM} = K.E_{rotation} + K.E_{translation}$;

$K.E = \frac{1}{2}I_{cmw^2} + \frac{1}{2}MV_{cm}^2$;

$K.E = \frac{1}{2}MK^2w^2 + \frac{1}{2}MV_{cm}^2$

→ angular momentum of Rigid body per forming CRTM: Pure Rotational as a Rigid body about C.O.M: Translation as a particle

(1) ROLLING ON INCLINED PLANE

$(E_k)_r$ = rotational K.E $(E_k)_t$ = translation K.E

(a) for solid sphere, $(E_k)_r$ = 40% of $(E_k)_t$,

(b) For snell $(E_k)_r$ = 66% of $(E_k)_t$,

(c) For disc, $(E_k)_r$ = 50% of $(E_k)_t$ of $(E_k)_t$,

(d) For ring, $(E_k)_r = (E_k)_t$

(2) VELOCITY AT LOWEST POINT

$$V = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}$$

(3) ACCELERATION ALONG INCLINED PLANE

$$a = \frac{g\sin\theta}{\sqrt{1+\frac{K^2}{R^2}}}$$

System of Particles and Rotational Dynamics

A group of particles that can be identified and distinguished from other particles or groups is called a system. The motion of the system as a whole can be analysed by applying the laws of mechanics. This is achieved by using the concept of 'centre of mass'. Thus, identifying the centre of mass is quite important in the study of dynamics of a system.

Rigid body: It is a body whose shape and size do not change during its state of rest or motion.

A classic example of a physical system is that of a rigid body, in which the relative distance between any two particles remains unaltered during its motion. Study of rigid body motion involves physical parameters like moment of inertia, torque, angular velocity, angular momentum, translational energy and rotational energy.

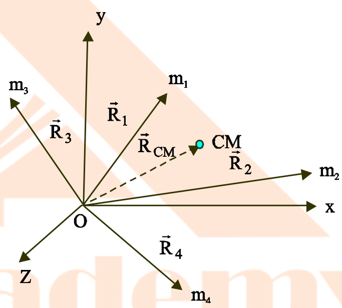
Centre of mass of a system of two particles

Centre of mass is the point at which the entire mass of a body is supposed to be concentrated.

If we have discrete system of particles as shown in the figure, then centre of mass is defined as

$$R_{cm} = \frac{m_1\vec{R}_1 + m_2\vec{R}_2 + m_3\vec{R}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

$$R_{cm} = \frac{1}{M}\sum m_i\vec{R}_i$$



The coordinates of centre of mass

$$X_{cm} = \frac{m_1x_1 + m_2x_2 + \ldots}{m_1 + m_2 + m_3} = \frac{1}{M}\sum m_ix_i$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{1}{M}\sum m_iy_i$$

$$Z_{cm} = \frac{m_1z_1 + m_2z_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{1}{M}\sum m_iz_i$$

For a system having continuous distribution of the mass, the coordinates of cm are

$$X_{cm} = \frac{1}{M}\int xdm, \quad Y_{cm} = \frac{1}{M}\int y_{dm}, \quad Z_{cm} = \frac{1}{M}\int zdm$$

- Position of centre of mass is independent of coordinate system choosen.
- Centre of mass depends on the shape of the body and distribution of mass.
- Centre of mass coincides with geometric centre bodies where mass is homogeneous.
- Centre of mass remains unchanged in rotatory motion while in translatory motion position changes.
- If small position of mass m_2 is removed from a larger position of mass m_1. Then centre of mass of the remaining part is $x_{cm} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}$

Motion of centre of mass

If a system of particles of masses m_1, m_2, . . move with velocities v_1, v_2, v_3 . . . respectively

- Then velocity of centre of mass is given by

$$V_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n}{m_1 + m_2 + \ldots + m_n} = \frac{\sum m_1\vec{v}_i}{M}$$

(M → total mass of the body)

- Momentum of the centre of mass is

$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2 + \vec{v}_2 + ... + m_n\vec{v}_n$

$\vec{P}_{CM} = \vec{P}_1 + \vec{P}_2 + ... + \vec{P}_n$

If $\vec{V}_{CM} = 0$, $\vec{P}_{CM} = 0$ i.e., in the frame of reference of CM, the momentum of a system is zero.

- Acceleration of CM is given by

$$\vec{a}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ... m_n\vec{a}_n}{m_1 + m_2 + ... m_n} = \frac{\sum m_i a_i}{M}$$

Or $M\vec{a}_{CM} = m_1\vec{a}_1 + m_2\vec{a}_2 + ... m_n\vec{a}_n$

$\vec{F}_{ext} = \vec{F}_1 + \vec{F}_2 + ... + \vec{F}_n$

This is equation of motion of centre of mass.

If $\vec{F}_{ext} = 0 \Rightarrow \vec{a}_{CM} = 0 \Rightarrow \vec{v}_{CM} =$ constant.

If $\vec{F}_{ext} = 0$, no external force acts on a system, then the velocity of its CM remains constant.

$\Rightarrow$ velocity of CM is not affected by internal forces.

- If $\vec{F}_{ext} = 0 \Rightarrow \vec{a}_{CM} = 0 \Rightarrow \vec{V}_{CM} =$ constant, then $\vec{p}$ = constant.

This leads to conservation of linear momentum.

- If a system of 2 particles of mass m_1 and m_2 separated by a distance x initially at rest, moving towards each other under the action of attractive force then the 2 particles collide at their centre of mass.

Here $F_{12} = -F_{21}$ or $m_1a_1 = m_2a_2 \Rightarrow \frac{a_1}{a_2} = \frac{m_2}{m_1}$

Since initial momentum = 0, centre of mass is at rest.

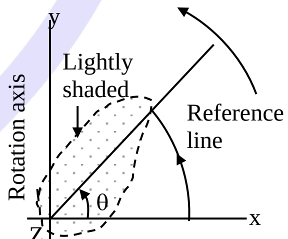
$\vec{V}_{CM} = 0$, $m_1v_1 = m_2v_2$ or $\frac{v_1}{v_2} = \frac{m_2}{m_1}$

$\therefore$ Ratio of distances covered by particles before collision is $\frac{x_1}{x_2} = \frac{m_2}{m_1}$

Rotational motion: A rigid body undergoes rotational motion when each of its particles travel in a circle centered on a straight line, called the axis of rotation

Rotational variables: The rotational variables are the angular equivalents of the linear quantities position, displacement, velocity and acceleration

Angular position (θ): It is the position of a fixed line perpendicular to the axis of rotation, fixed in the body, relative to a fixed axis. It is also called the angular coordinate. $\theta = \frac{s}{r}$ where



A rigid body rotating anticlockwise in the xy- phase about z axis the rotation axis

s → arc length described by a point on the reference line relative to the fixed axis.

r → radius of the arc. Its SI unit is the radian (rad) without any dimensions.

Angular displacement: It is the difference in the angular coordinates of rotating body at times t_1 and $t_2 = t_1 + \Delta t$

$\Delta\theta = \theta_2 - \theta_1$

Angular velocity

(i) **Average angular velocity (ω_{av}):** It is the ratio of the angular displacement to the elapsed time

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

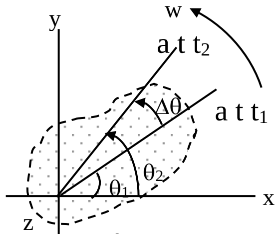
(ii) **Angular velocity(ω):** It is the rate of change of angular displacement

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

It is the limiting value of the average angular velocity

The SI unit of angular velocity is $rad.s^{-1}$.

Its dimensional formula is $[M^{\circ} L^{\circ} T^{-1}]$



Uniform circular motion

Angular acceleration

(i) **Average angular acceleration(α_{av}):** It is the ratio of the change in angular velocity to the elapsed time

$$\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

(ii) **Angular acceleration(α):** It is the rate of change of angular velocity

$$\alpha = \lim_{\Delta t \to o} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

It is the limiting value of the average angular acceleration.

Its SI unit is $rad\ s^{-2}$

Its dimensional formula in $[M^{\circ} L^{\circ} T^{-2}]$

- Finite angular displacements are not vectors since they do not obey commutative law
- Infinitesimal (differential) angular displacements are, however, treated as vectors.
- The direction of rotational variables specifies the axis of rotation, and is given by the right hand rule
- Nothing moves in the direction of angular variables, i.e., along the axis of rotation.

Relations connecting linear and angular variables:

(a) $s = r\theta$; (b) $v = \omega r$; (c) $a = \alpha r$

The following table contains linear as well as angular equations of motion for constant acceleration

Angular			Linear		
Sl.no.	Equation	Missing quantity	Sl.no.	Equation	Missing quantity
1	$\omega = \omega_o + \alpha t$	θ	1	$v = u + at$	s
2	$\theta = \omega_o t + \frac{\alpha t^2}{2}$	ω	2	$s = ut + \frac{at^2}{2}$	v
3	$\omega^2 = \omega_o^2 + 2\alpha\theta$	t	3	$\omega^2 = u^2 + 2as$	t
4	$\theta = \omega_{av} t = \frac{(\omega + \omega_o)t}{2}$	α	4	$s = u_{av}\, t = \frac{(u+v)t}{2}$	a
5	$\theta_n = \omega_o + \frac{\alpha(2n-1)}{2}$	ω	5	$s_n = u + \frac{a(2n-1)}{2}$	v
6	$\alpha = \frac{\theta_2 - \theta_1}{t^2}$	ω_o, ω	6	$a = \frac{s_2 - s_1}{t^2}$	u, v

Moment of inertial (I):

a. Moment of inertia of a particle about an axis is defined as the product of its mass m and square of its distance r from the axis of rotation.

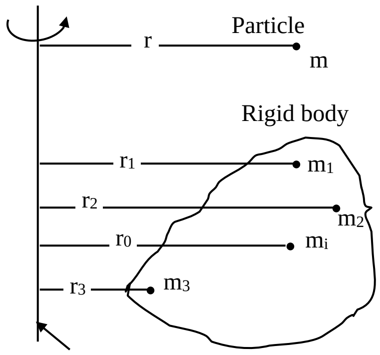
$I = mr^2$

b. Moment of inertia of a rigid body about an axis is the sum of the moments of inertia of its constituent particles about that axis

$$I = \sum_{i=1}^{i=n} m_i r_i^2 \quad \text{for a rigid body}$$

The SI unit of moment of inertia is $kg.m^2$

The dimensional formula for I is $[M^1L^2T^\circ]$



Axis of rotation. Moment of inertia of a particle and a rigid body about an axis

(i) Moment of inertia of a body changes if the axis of rotation is changed.

(ii) Moment of inertia of a body depends on (a) the mass, (b) the distance and (c) the manner in which mass of the body is distributed.

(iii) It plays the same role in rotational motion as mass does in translational motion. So, it is the counter part of mass.

(iv) Moment of inertia of continuous and homogeneous mass distribution about an axis is given by

$$I = s\int_v r^2\, dv$$

where s $\rightarrow$ mass density of the material

r $\rightarrow$ distance of the differential volume element from the axis of rotation

$\int_v$ $\rightarrow$ integration over the volume occupied by the mass

Radius of gyration (k): It is the distance of the centre of mass of a body about its axis of rotation such that the M.I. of the centre of mass equals that of the body. It is given by $k = \sqrt{\frac{I}{M}}$

SI unit of radius of gyration is the metre.

The dimensional formula for radius of gyration is $[M^\circ L^1 T^\circ]$

The M.I. of a body about an axis is that property of the body that causes it to resist a change in its angular velocity about that axis.

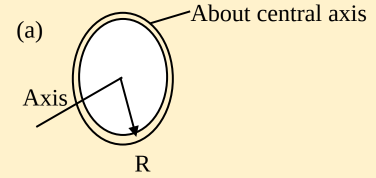
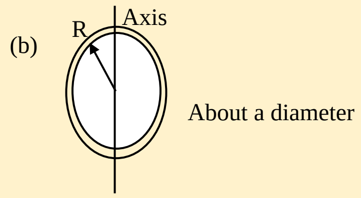
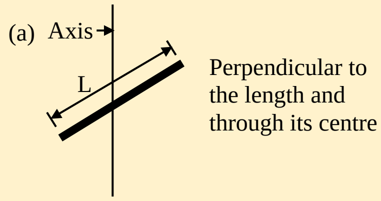
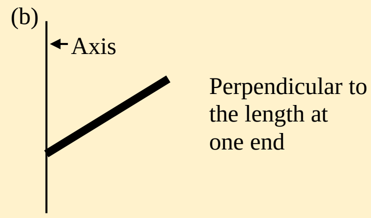
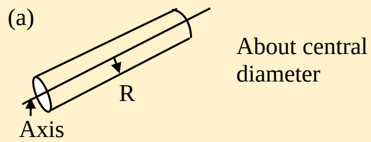
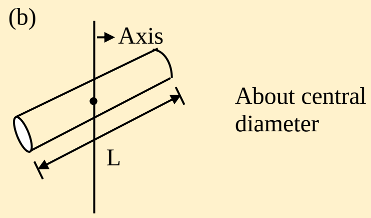
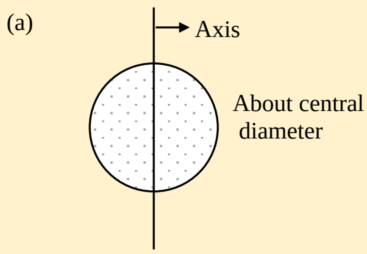
Perpendicular axis theorem: The moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the lamina about two mutually perpendicular axes in the plane of the lamina, all three axes originating at the same point.

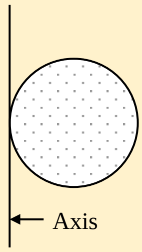
$I_z = I_x + I_y$

Parallel axes theorem: The moment of inertia of a body about any axis is equal to the sum of its moments of inertia about a parallel axis through its centre of mass and the product of its mass and square of the distance between the two axes.

$I = I_c + mr^2$

M.I. of some rigid bodies

Sl. No	Object	Axis of rotation	Expression for MI I	Radius of gyration k
1	Hoop (Thin circular ring)		MR^2	R
			$\frac{MR^2}{2}$	$\frac{R}{\sqrt{2}}$
2	Thin rod		$\frac{ML^2}{12}$	$\frac{L}{\sqrt{12}}$
			$\frac{ML^2}{3}$	$\frac{L}{\sqrt{3}}$
3.	Solid cylinder (or disc)		$\frac{MR^2}{2}$	$\frac{R}{\sqrt{2}}$
			$M\left[\frac{R^2}{4}+\frac{L^2}{12}\right]$	$\sqrt{\frac{R^2}{4}+\frac{L^2}{12}}$
4	Solid sphere		$\frac{2MR^2}{5}$	$\sqrt{\frac{2}{5}}R$

Sl. No	Object	Axis of rotation	Expression for MI I	Radius of gyration k
		(b)  About a tangent	$\frac{7MR^2}{5}$	$\sqrt{\frac{7}{5}}R$

Angular momentum (L): It is the counter part of linear momentum. Angular momentum of a particle about an axis is defined as

$$\vec{L}=\vec{r}\times\vec{p}=rmv\sin\theta\cdot\hat{n}$$

where $\vec{r}$ $\rightarrow$ position vector relative to O

$\vec{v}$ $\rightarrow$ velocity of the particle of mass m

$\hat{n}$ $\rightarrow$ unit vector in the direction of $\vec{L}$

$\theta \rightarrow$ angle between $\vec{r}$ and $\vec{p}$ or $\vec{v}$

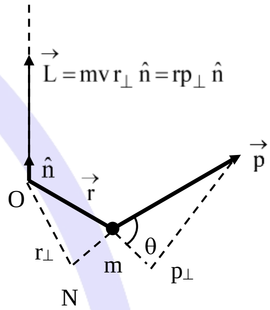


Illustration of angular momentum

The direction of $\vec{L}$ is always perpendicular to the plane formed by $\vec{r}$ and $\vec{p}$, the sense being given by right hand grasp rule.

The SI unit of angular momentum is kg m^2 s^{-1}.

The dimensional formula for angular momentum is $[M L^2 T^{-2}]$

Torque ($\vec{\tau}$) : It is the counter part of force. Moment of a force, i.e., torque, is defined as

$$\vec{\tau}=\vec{r}\times\vec{F}=rF\sin\theta\,\hat{n}$$

When $\vec{r}$ $\rightarrow$ position vector of a particle of mass m relative to O,

$\vec{F}$ $\rightarrow$ force acting on the particle

$\hat{n}$ $\rightarrow$ unit vector in the direction of $\vec{\tau}$

$\theta \rightarrow$ angle between $\vec{r}$ and $\vec{F}$

The SI unit of torque is N m

The dimensional formula for torque is $[M^1L^2T^{-2}]$

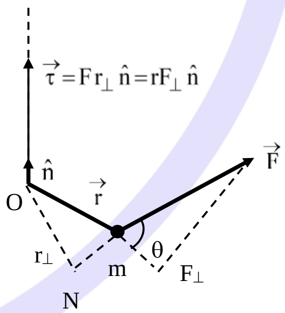


Illustration of torque

- The turning or the rotating effect of a force or a couple about an axis is measured by the torque.
- No torque acts on a body rotating with a constant angular speed about an axis.
- Torque and work are two different quantities, though they have the same unit Nm. Work is expressed in joule, but there is no specific unit for torque.

Torque and angular momentum: Torque is the rate of change of angular momentum.

$$\vec{\tau}=\frac{d\vec{L}}{dt}=\frac{d}{dt}[m(\vec{r}\times\vec{v})]=m\left[\frac{d\vec{r}}{dt}\times\vec{v}+\vec{r}\times\frac{d\vec{v}}{dt}\right]$$

$$=m[\vec{v}\times\vec{v}]+m\,\vec{r}\times\vec{a}=\vec{r}\times(m\vec{a})\ (\because \vec{v}\times\vec{v}=0)\quad=\vec{r}\times\vec{F}$$

Relation connecting moment of inertia and

(a) angular momentum : L = Iω

(b) torque: $\tau = I\alpha$

(c) rotational kinetic energy: $K_r = \frac{I\omega^2}{2}$

Law of conservation of angular momentum: In the absence of external torque on a system, the angular momentum of that system remains constant, no matter what changes take place with the system.

$\vec{L} = \text{constant}$, or $\Delta\vec{L} = 0$

or $I_1\omega_1 = I_2\omega_2 = \text{constant}$

or $m_1\omega_1 r_1^2 = m_2\omega_2 r_2^2 = \text{constant}$

or $m_1v_1r_1 = m_2v_2r_2 = \text{constant}$

(all) If $\tau = \frac{dL}{dt} = 0$

☞

- The angular speed of a spinning volunteer increases when the volunteer pulls his/her out stretched hands. This is due to the law of conservation of angular momentum. MI of the volunteer about the axis is more when the hands are stretched and less when the hands are pulled.
- A spring board diver leaves the board with a definite angular momentum about a horizontal axis perpendicular to the vertical plane containing the parabolic path of the diver. The diver changes from **open layout position** to **closed position** and vice versa so as to conserve angular momentum during somersault.

Comparison of rotational and translational quantities

Sl. No.	Physical quantity	Pure translation (fixed direction)	Pure rotation (fixed)
1	Position coordinate	x	θ
2	Velocity	$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$
3	Acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$
4	Cense for motion (Newton 2nd law)	$F = ma$ (force)	$\tau = I\alpha$
5	Work	$w = \int F dx$	$\int \tau\, d\theta$
6	Kinetic energy	$k = \frac{mv^2}{2}$	$k_r = \frac{I\omega^2}{2}$
7	Power	$P = Fv$	$P_r = \tau\omega$
8	Momentum	$p = mv$	$L = I\omega$
9	Inertia	Mass m	Moment of inertia $I = mr^2$
10	Work energy theorem	$W = k_f - k_i$	$W = k_{rf} - k_{ri}$
11	Law of conservation of momentum	$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$ or mv = constant	$\Delta\vec{L} = \vec{L}_f - \vec{L}_i$ or $I\omega$ = constant

Kinetic energy of rolling motion

Kinetic energy of rolling motion can be separated into kinetic energy of translation and kinetic energy of rotation.

i.e., k = k (translation) + k(rotation)

$$= \frac{mv^2}{2} + \frac{1}{2}I\omega^2$$

Put $I = mk^2$, where k is radius of gyration of body and $v_c = R\omega$

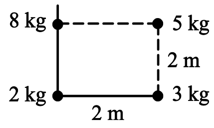
$$\therefore\ k = \frac{1}{2}\frac{mk^2v_{cm}^2}{R^2} + \frac{1}{2}mv_{cm}^2 \text{ or } k = \frac{1}{2}mv_{cm}^2\left[1 + \frac{k^2}{R^2}\right]$$

Illustrations

1. Four bodies of masses 2, 3, 5 and 8 kg are placed at the four corners of a square of side 2 m. The position of CM will be

(A) $\left(\frac{8}{9}, \frac{13}{9}\right)$m (B) $\left(\frac{7}{9}, \frac{11}{9}\right)$m

(C) $\left(\frac{11}{9}, \frac{13}{9}\right)$m (D) $\left(\frac{11}{9}, \frac{8}{9}\right)$m



Ans (A)

The co-ordinates of the corners of the square are (0, 0), (2, 0), (2, 2), (0, 2). Hence,

$$x_{COM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4} = \frac{2\times0 + 3\times2 + 5\times2 + 8\times0}{2+3+5+8} = \frac{16}{18} = \frac{8}{9}m$$

$$y_{COM} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4} = \frac{2\times0 + 3\times0 + 5\times2 + 8\times2}{2+3+5+8} = \frac{26}{18} = \frac{13}{9}m$$

$\therefore$ Coordinates of the centre of mass $= \left(\frac{8}{9}, \frac{13}{9}\right)$m

2. Two bodies of mass 10 kg and 2 kg are moving with velocities $2\hat{i} - 7\hat{j} + 3\hat{k}$ and $-10\hat{i} + 35\hat{j} - 3\hat{k}$ ms^{-1} respectively. The velocity of their centre of mass is

(A) $2\hat{i}$ ms^{-1} (B) $2\hat{k}$ ms^{-1} (C) $(2\hat{j} + 2\hat{k})$ ms^{-1} (D) $(2\hat{i} + 2\hat{j} + 2\hat{k})$ ms^{-1}

Ans (B)

m_1 = 10 kg, m_2 = 2 kg

$\vec{v}_1 = 2\hat{i} - 7\hat{j} + 3\hat{k}$

$\vec{v}_2 = -10\hat{i} + 35\hat{j} - 3\hat{k}$

$$\vec{v}_{COM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \frac{10(2\hat{i} - 7\hat{j} + 3\hat{k}) + 2(-10\hat{i} + 35\hat{j} - 3\hat{k})}{10+2} = 2\hat{k} \text{ ms}^{-1}$$

3. A mass of 1 kg is placed at (1m, 2m, 0). Another mass of 2 kg is placed at (3m, 4m, 0). Find the moment of inertia of the system of these masses about z-axis.

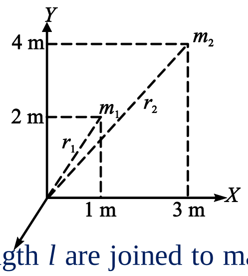
(A) 18.33 kg m^2 (B) 55 kg m^2 (C) 44 kg m^2 (D) 50 kg m^2

Ans (B)

m_1 = 1 kg, $r_1 = \sqrt{1^2 + 2^2} = \sqrt{5}$ m

m_2 = 2 kg, $r_2 = \sqrt{3^2 + 4^2} = 5$ m

$I_z = (1)\left(\sqrt{5}\right)^2 + 2(5)^2 = 55$ kg m^2



4. Four thin rods each of mass m and length l are joined to make a square. Find the moment of inertia of all the four rods about any side of the square.

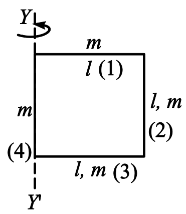
(A) $\frac{2}{3}ml^2$ (B) $\frac{2}{3}ml^2$ (C) 2 ml^2 (D) $\frac{5}{3}ml^2$

Ans (D)

$I_{yy'} = I_1 + I_2 + I_3 + I_4$

$= \frac{ml^2}{3} + ml^2 + \frac{ml^2}{3} + 0$

$= \frac{5ml^2}{3}$



5. Moment of inertia of a uniform rod of mass m and length l is $\frac{17}{12}ml^2$ about a line perpendicular to the rod. Find the distance of this line from the middle point of the rod.

(A) l (B) $\frac{l}{2\sqrt{3}}$ (C) $\frac{l}{\sqrt{2}}$ (D) $\frac{2}{3}l$

Ans (C)

$$I = I_{COM} + mr^2 \Rightarrow \frac{7}{12}ml^2 = \frac{ml^2}{12} + mr^2$$

$$\therefore\ r^2 = \frac{l^2}{2} \Rightarrow r = \frac{l}{\sqrt{2}}$$

6. Radius of gyration of a body about an axis at a distance 6 cm from its centre of mass is 10 cm. Find its radius of gyration about a parallel axis through its centre of mass.

(A) 6 cm (B) 8 cm (C) 10 cm (D) 12 cm

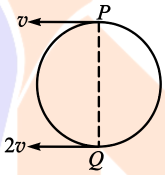
Ans (B)

$$I = I_{COM} + mr^2 \Rightarrow m(10)^2 = m(k)^2 + m(6)^2 \quad \therefore\ k = 8 \text{ cm}$$

7. Two points P and Q, diametrically opposite on a disc of radius R have linear velocities v and 2 v as shown, in figure. Find the angular speed of the disc.

(A) $\frac{3v}{2R}$ (B) $\frac{v}{2\sqrt{2}R}$

(C) $\frac{v}{R}$ (D) $\frac{v}{2R}$



Ans (D)

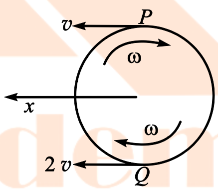
Let the velocity of centre is x and angular speed is ω

At $P: x - R\omega = v$

At $Q: x + R\omega = 2v$ Subtracting

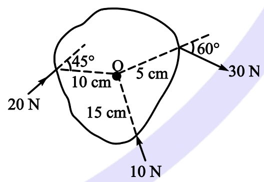
$-2r\omega = -v$

$\omega = \frac{v}{2R}$



8. Point O is the centre of mass of the rigid body shown in figures. Find the magnitude of torque (in N m) of the rigid body about point O when forces applied are as shown.

(A) 2.71
(B) 4.52
(C) 7.22
(D) 1.23



Ans (A)

$\tau_{10N} = 0$

$\tau_{20N} = (20\cos 45° \times 0.1)$ (clockwise)

$= 1.414$ Nm

$\tau_{30N} = (30\sin 60° \times 0.05)$ (clockwise)

$= 1.299$ Nm

$\therefore \tau_{Total} = 2.71$ Nm

9. A body rotating at 20 rad s^{-1} is acted upon by a constant torque providing it a deceleration of 2 rad s^{-2}. At what time will the body have kinetic energy same as the initial value if the torque continues to act?

(A) 10 s (B) 20 s (C) 15 s (D) 30 s

Ans (B)

$\omega_0 = 20$ rad s^{-1}; $\alpha = -2$ rad s^{-2}, $\omega = 0$

$$\omega = \omega_0 + \alpha t_1 \Rightarrow t_1 = \frac{20}{2} = 10\text{s}$$

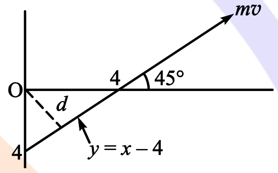
At $t_1 = 10$, body comes to a halt. If the torque continues to act the body acquires initial angular speed in the opposite direction (i.e., initial kinetic energy) after another 10 s.

∴ Required time is $t = 10 + 10 = 20$ s

10. A particle of mass m moves in xy plane along the line $y = x - 4$, with constant speed v. Find the angular momentum of the particle about origin at any instant of time.

(A) mv (B) $2mv$ (C) $\frac{2}{\sqrt{3}}mv$ (D) $2\sqrt{2}\,mv$

Ans (D)



Perpendicular distance d from the origin is $d_\perp = 4\sin 45° = 2\sqrt{2}$ m

$\therefore\ L = mvd_\perp = 2\sqrt{2}\,mv$

11. The centre of mass of a system of particles does not depend on

(A) masses of the particles (B) forces acting on the particles

(C) position of the particles (D) relative distances between the particles

Ans (B)

The resultant of all the forces on any system of particles is zero. Therefore, their centre of mass does not depend upon the forces acting on the particles.

12. The angular momentum of a 10 g particle moving with velocity $5\hat{i}$ m s^{-1} and having position vector $(10\hat{i} + 6\hat{j})$ metre about the origin is

(A) $-0.1\ \hat{k}$ Js (B) $-0.2\ \hat{k}$ Js (C) $-0.3\ \hat{k}$ Js (D) $-0.4\ \hat{k}$ Js

Ans (C)

Angular momentum $\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$

$$= \frac{10}{1000}[(10\hat{i} + 6\hat{j}) \times 5\hat{i}]$$

$$= 0.01(30\ \hat{j} \times \hat{i})$$

$$= -0.3\hat{k}\text{ Js}\ \ (\because \hat{i} \times \hat{i} = 0\ \&\ \hat{j} \times \hat{i} = -\hat{k})$$

13. The ratio of the angular speeds of the hour hand to the minute hand of a clock is

(A) 1 : 1 (B) 43200 : 1 (C) 720 : 1 (D) 1 : 12

Ans (D)

Angular velocity $\omega = \frac{\theta}{t}$

$$\frac{\text{Angular velocity of the hour hand}(\omega_1)}{\text{Angular velocity of the minute hand}(\omega_2)} = \frac{\theta / t_1}{\theta / t_2}$$

$$= \frac{t_2}{t_1} = \frac{1\text{ hr}}{12\text{ hr}} = \frac{1}{12}$$

14. Two bodies are rotating about an axis, their angular momentum being the same. Moment of inertia of body –1 is I_1 and that of body –2 is I_2. Their respective rotational kinetic energies are K_1 and K_2.

If $I_1 > I_2$, then

(A) $K_1 > K_2$

(B) $K_1 < K_2$

(C) $K_1 = K_2$

(D) the data is insufficient to predict whether $K_1 > K_2$, $K_2 > K_1$, or $K_1 = K_2$.

Ans (B)

The kinetic energy of a rotating body about an axis is $K = \frac{1}{2}I\omega^2$

Let ω_1 and ω_2 be the angular speeds of the bodies 1 and 2 respectively. The kinetic energy of body –1 and body – 2 are respectively

$K_1 = \frac{1}{2} I_1\omega_1^2$ and $K_2 = \frac{1}{2} I_2 \omega_2^2$. But the angular momentum is same for both the bodies,

i.e. $L = I_1\omega_1 = I_2\omega_2$.

$$\frac{K_1}{K_2} = \frac{I_1\omega_1^2}{I_2\omega_2^2} = \frac{(I_1\omega_1)^2/I_1}{(I_2\omega_2)^2/I_2} = \frac{I_2}{I_1} < 1 \quad \text{since } I_1 > I_2$$

$\therefore K_1 < K_2$

15. A body rolls without slipping. The radius of gyration of the body about an axis passing through its centre of mass is k. The radius of the body is R. The ratio of rotational kinetic energy to translational kinetic energy is

(A) $\frac{k^2}{R^2}$ (B) $\frac{R^2}{k^2+R^2}$ (C) $\frac{k^2}{k^2+R^2}$ (D) $k^2 + R^2$

Ans (A)

Rotational kinetic energy is $K_r = \frac{I\omega^2}{2} = \frac{mk^2v^2}{2R^2}$

Translational kinetic energy is $K = \frac{mv^2}{2}$ $\therefore \frac{K_r}{K} = \frac{k^2}{R^2}$

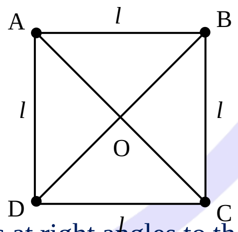
16. There are four point masses m each on the corners of a square of side length l. What is the moment of inertia of the system about one of its diagonals?

(A) $2ml^2$ (B) ml^2 (C) $4ml^2$ (D) $6ml^2$

Ans (B)

MI about one of its diagonals $= m(AO)^2 + m(OC)^2$

$$= m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 = ml^2$$



17. The moment of inertia of a meter stick of mass 300 gm, about an axis at right angles to the stick and located at 30 cm mark, is

(A) 8.3×10^5 g cm^2 (B) 5.8 g cm^2 (C) 3.7×10^5 g cm^2 (D) none of these

Ans (C)

According to theorem of parallel axes

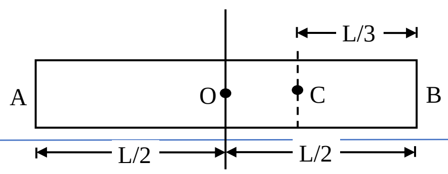
$$I = I_{CG} + Md^2 = \frac{Ml^2}{12} + Md^2 = 300\left[\frac{100^2}{12} + 20^2\right] = 3.7 \times 10^5 \text{ gm cm}^2$$

18. The moment of inertia of a uniform thin rod of length L and mass M about an axis passing through a point at a distance of $\frac{L}{3}$ from one of its ends and perpendicular to the rod is

(A) $\frac{7ML^2}{48}$ (B) $\frac{ML^2}{1}$ (C) $\frac{ML^2}{9}$ (D) $\frac{ML^2}{3}$

Ans (C)

The distance $OC = \frac{L}{2} - \frac{L}{3} = \frac{L}{6}$

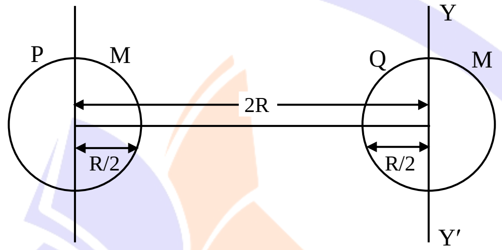


Applying the theorem of parallel axes,

$I_C = I_O + M(OC)^2$

$$\frac{ML^2}{12} + M\left(\frac{L}{6}\right)^2 = \frac{ML^2}{9}$$

19. Two spheres each of mass M and radius $\frac{R}{2}$ are connected with a massless rod of length 2R as shown in the figure. The moment of inertia of the system about an axis passing through the centre of one of the sphere and perpendicular to the rod is



(A) $\frac{21}{5}MR^2$

(B) $\frac{2}{5}MR^2$

(C) $\frac{5}{2}MR^2$

(D) $\frac{5}{21}MR^2$

Ans (A)

According to theorem of parallel axes,

$$I = \frac{2}{5}M\left(\frac{R}{2}\right)^2 + M(2R)^2 + \frac{2}{5}M\left(\frac{R}{2}\right)^2 = 4MR^2 + \frac{1}{5}MR^2 = \frac{21}{5}MR^2$$

20. The moment of inertia of a body about a given axis is 1.2 kg m^2. Initially, the body is at rest. In order to produce a rotational KE of 1500 joule, an angular acceleration of 25 rad/sec^2 must be applied about that axis for a duration of

(A) 4 s (B) 2 s (C) 8 s (D) 10 s

Ans (B)

$$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}(\alpha t)^2 = \frac{1}{2}I^2 \propto t^2$$

$$1500 = \frac{1}{2}\times 1.2\times(25)^2 t^2 \quad \text{or} \quad t^2 = 4 \qquad \text{or} \quad t = 2\text{ s}$$

21. A flywheel of radius 2 m and mass 8 kg rotates at an angular speed of 4 rad/s about an axis perpendicular to it through its centre. The kinetic energy of rotation is

(A) 128 J (B) 196 J (C) 256 J (D) 392 J

Ans (A)

M = 8 kg R = 2 m ω = 4 rad/sec

$$(KE)_{Rotation} = \frac{1}{2}I\omega^2 \quad \text{or} \quad K_R = \frac{1}{2}\times\frac{1}{2}MR^2\times\omega^2 = \frac{1}{4}\times 8\times 4\times 16 = 128\text{ J}$$

22. A constant torque of 31.4 N m is exerted on a pivoted wheel. If angular acceleration of wheel is 4π rad/sec^2, then the moment of inertia of the wheel is

(A) 2.5 kg m^2 (B) 3.5 kg m^2 (C) 4.5 kg m^2 (D) 5.5 kg m^2

Ans (A)

$\tau = 31.4 = I\alpha = I \times 4\pi$

$$\therefore \quad I = \frac{31.4}{4\pi} = 2.5\ \text{kg m}^2$$

23. The ratio of the radii of gyration of a circular disc and a circular ring of the same radii about a tangential axis is

(A) $1:\sqrt{2}$ (B) $\sqrt{5}:\sqrt{6}$ (C) $\sqrt{2}:\sqrt{3}$ (D) $\sqrt{2}:1$

Ans (B)

$$I_d = \frac{1}{4}m_1r_1^2 + m_1r_1^2 = \frac{5}{4}m_1r_1^2 \text{ and } I_r = \frac{1}{2}m_2r_2^2 + m_2r_2^2 = \frac{3}{2}m_2r_2^2$$

But $r_1 = r_2$.

$$\therefore K_d^2 / K_r^2 = \frac{5}{4} / \frac{3}{2} = \frac{5}{6} \Rightarrow \frac{K_d}{K_r} = \frac{\sqrt{5}}{\sqrt{6}}.$$

24. Rotational motion can be
(A) one or two dimensional
(B) two or three dimensional
(C) three or one dimensional
(D) none of the above

Ans (B)
Rotational motion cannot be one dimensional.

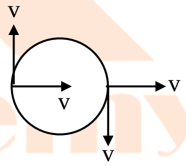
25. A flywheel rotates at a constant acceleration of 2 rad s^{-2}. If the wheel starts from rest, the number of revolutions that it makes in first ten seconds is approximately
(A) 16
(B) 24
(C) 32
(D) 8

Ans (A)

$\theta = \omega_i t + \frac{1}{2}\alpha t^2$ But $\omega_i = 0 \Rightarrow \theta = \frac{1}{2} \times 2 \times 10^2 = 100$ rad

Number of rotations $n = \frac{\theta}{2\pi} = \frac{100}{2\pi} \approx 16$.

26. The center of a wheel rolling on a plane surface moves with a speed v_0. A particle on the rim of the wheel at the same level as the center will be moving at a speed



(A) zero
(B) v_0
(C) $\sqrt{2}\ v_0$
(D) $2\ v_0$

Ans (C)
In pure rolling, $v = r\omega$
Speed of the points at same height as that center is $\sqrt{v^2 + v^2} = \sqrt{2}\,v_0$

27. Four spheres, each of mass M and diameter 2r, are placed with their centres on the corners of a square of side a (a > 2r). The moment of inertia of the system about one side of the square is
(A) $\frac{2}{5}$ M $(5r^2 + 4a^2)$
(B) $\frac{2}{5}$ M $(5r^2 + 2a^2)$
(C) $\frac{2}{5}$ M $(2r^2 + 5a^2)$
(D) $\frac{2}{5}$ M $(4r^2 + 5a^2)$

Ans (D)

$$I = 2\left(\frac{2}{5}Mr^2\right) + 2\left(\frac{2}{5}Mr^2 + Ma^2\right) = \frac{2}{5}(4Mr^2 + 5Ma^2) = \frac{2}{5}M(4r^2 + 5a^2).$$

28. Two discs A and B have same mass and same thickness but A is made of aluminium and B of lead. About the central axis, the moment of inertia of
(A) both the discs are the same
(B) disc A is larger
(C) disc B is larger
(D) the two discs cannot be same

Ans (B)
In the case of aluminium, density is less. The distribution of mass is farther from the axis.

29. Two circular loops A and B of radii R and 2R respectively are made of the same wire. Their moments of inertia about the axis passing through the centre and perpendicular to their plane are I_A and I_B respectively. The ratio I_A/I_B is
(A) 1
(B) $\frac{1}{2}$
(C) $\frac{1}{4}$
(D) $\frac{1}{8}$

Ans (D)

Let μ be the mass per unit length of the wire.

$M_A = 2\pi R\mu$ and $M_B = 4\pi R\mu$.

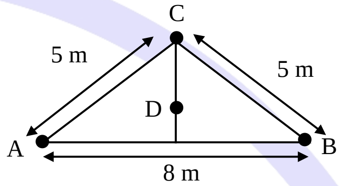
$I_A = M_A R_A^2 = 2\pi R\mu \times R^2 = 2\pi\mu R^3$

and $I_B = M_B R_B^2 = 4\pi R\mu \times (2R)^2 = 16\pi\mu R^3$

$\therefore \frac{I_A}{I_B} = \frac{1}{8}$

30. Three particles each of mass 1 kg are placed at the vertices of a triangle. Fourth particle of mass 2 kg is placed at the centroid of the triangle. Find the moment of inertia about the side AB of the triangle.

(A) 11 kg m^2
(B) 21 kg m^2
(C) 201 kg m^2
(D) 121 kg m^2



Ans (A)

The particles A and B are on the axis. The perpendicular distances of the particles C(1 kg) and D(2 kg) are 3 m and 1 m from the axis AB.

∴ Moment of inertia of four particle system is $I = \sum m_i r_i^2 = 1(C)^2 + 2(1)^2 = 11$ kg m^2

31. A balance is made of a rigid rod free to rotate about a point not at the centre of the rod. When an unknown mass m is placed in the left-hand pan, it is balanced by a mass m_1 placed in the right-hand pan and similarly when the mass m is placed in the right-hand pan, it is balanced by a mass m_2 in the left-hand pan. Neglecting the masses of the pans, m is

(A) $\frac{m_1 + m_2}{2}$ (B) $\sqrt{m_1 m_2}$ (C) $\frac{\sqrt{m_1^2 + m_2^2}}{2}$ (D) $\sqrt{\frac{m_1^2 + m_2^2}{2}}$

Ans (B)

Torque about support is zero.

$mgx = m_1gy$ … (1)

$m_2gx = mgy$ … (2)

From the above equations, we get $m = \sqrt{m_1 m_2}$

32. A wheel of mass 40 kg and radius of gyration 0.5 m comes to rest from a speed of 1800 revolution per minute in 30 s. Assuming that the retardation is uniform, the value of the retarding torque in N m, is

(A) 10π (B) 20π (C) 30π (D) 40π

Ans (B)

The retardation is given by zero = $\omega_0 - \alpha t$

$\omega_0 = \frac{1800 \times 2\pi}{60} = 60\pi$ rad s^{-1} ∴ $\alpha = \frac{\omega_0}{t} = 2\pi$ rad s^{-1}

Moment of inertia of the wheel is $I = Mk^2 = 40 \times 0.5^2 = 10$ kg m^2

Retarding torque = Iα = 20π N m.

33. A wheel with moment of inertia 2 kgm^2 about its axis, rotates at 50 rpm. The torque that can stop the wheel in one minute is

(A) $\frac{\pi}{9}$ N m (B) $\frac{\pi}{18}$ N m (C) $\frac{\pi}{36}$ N m (D) π N m

Ans (B)

The initial angular velocity $= 50\,\text{rpm} = \frac{5\pi}{3}\,\text{rad}\,s^{-1}$

Using $\omega = \omega_0 + \alpha t$, $\alpha = \frac{\omega - \omega_0}{t} = \frac{0 - \frac{5\pi}{3}}{60} = -\frac{\pi}{36}\,\text{rad}\,s^{-2}$

The torque that can produce this deceleration is $\tau = I\alpha = 2\times\frac{\pi}{36} = \frac{\pi}{18}$ N m .

34. When a mass is rotated in a plane about a fixed point, its angular momentum is directed along

(A) the radius (B) the tangent to the orbit

(C) a line at an angle of 45° to the plane of rotation (D) the axis of rotation

Ans (D)

The angular momentum $\vec{L} = \vec{r} \times m\vec{v}$ is perpendicular to both the radius vector and linear velocity of the particle.

35. A particle is moved in a circle with a constant angular velocity. Its angular momentum is $\vec{L}$. If the radius of the circle is halved keeping the angular velocity same, the angular momentum of the particle is

(A) $\frac{\vec{L}}{4}$ (B) $\frac{\vec{L}}{2}$ (C) $\vec{L}$ (D) $2\vec{L}$

Ans (A)

Given, $\vec{L} = mr^2\omega$ and $\vec{L}' = m\frac{r^2}{4}\omega$ Therefore, $\frac{\vec{L}'}{\vec{L}} = \frac{1}{4} \Rightarrow \vec{L}' = \frac{\vec{L}}{4}$.

36. A particle of mass m is projected with a velocity v making an angle of 45° with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection when the particle is at its maximum height h is

(A) zero (B) $\frac{mv^3}{4\sqrt{2}\,g}$ (C) $\frac{mv^3}{\sqrt{2}g}$ (D) $m\sqrt{2gh^3}$

Ans (B) and (D)

Maximum height attained is $h = \frac{v^2\sin^2 45°}{2g} = \frac{v^2}{4g}$

Momentum at the highest point is $p = mv\cos 45° = \frac{mv}{\sqrt{2}}$ (along horizontal)

Therefore, angular momentum about the origin is $\vec{L} = \frac{mv}{\sqrt{2}} \times h = \frac{mv^3}{4\sqrt{2}g}$

$\vec{L} = \frac{m}{\sqrt{2}}\times\sqrt{4gh}\times h = m\sqrt{2gh^3}$

37. A ballet dancer is spinning with a constant angular speed ω with her arms stretched out wards from the body. After a while, she gradually pulls her arms towards the body. Then

(A) the angular speed increases (B) the angular momentum increases

(C) the angular speed decreases (D) moment of inertia increases

Ans (A)

The angular momentum of the dancer Iω = constant. The moment of inertia decreases, when the dancer pulls her arms towards the body. Therefore, the angular speed increases in order to conserve the angular momentum.

38. A thin circular ring of mass M is rotating about its axis with a constant angular velocity ω. Two objects, each of mass m, are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity

(A) $\frac{\omega M}{M+m}$ (B) $\frac{\omega(M-2m)}{M+2m}$ (C) $\frac{\omega M}{M+2m}$ (D) $\frac{\omega(M+2m)}{M}$

Ans (C)

Angular momentum is conserved.

$I\omega = I'\omega'$

$MR^2\omega = (M + 2m)R^2\omega'$

$\omega' = \left(\frac{M}{M+2m}\right)\omega$

39. A constant torque acting on a uniform circular wheel changes its angular momentum from A_0 to $4A_0$ in 4 s. The magnitude of this torque is

(A) $\frac{3A_0}{4}$ (B) A_0 (C) $4A_0$ (D) $12A_0$

Ans (A)

Angular impulse = change in angular momentum

$\tau t = 4A_0 - A_0 \quad \tau = \frac{3A_0}{t} = \frac{3A_0}{4}$

40. Two bodies with moments of inertia I_1 and I_2 ($I_1 > I_2$) have equal angular momenta. If E_1 and E_2 are their rotational kinetic energies respectively, then

(A) $E_1 > E_2$

(B) $E_1 = E_2$

(C) $E_1 < E_2$

(D) the one which has larger mass has the larger kinetic energy

Ans (C)

Given $I_1\omega_1 = I_2\omega_2 \Rightarrow \frac{\omega_1}{\omega_2} = \frac{I_2}{I_1}$

$E_1 = \frac{1}{2} I_1\omega_1^2$ and $E_2 = \frac{1}{2} I_2\omega_2^2$ $\quad \therefore \frac{E_1}{E_2} = \frac{I_1\omega_1^2}{I_2\omega_2^2} = \frac{I_2}{I_1} < 1 \Rightarrow E_1 < E_2$

NCERT LINE BY LINE QUESTIONS

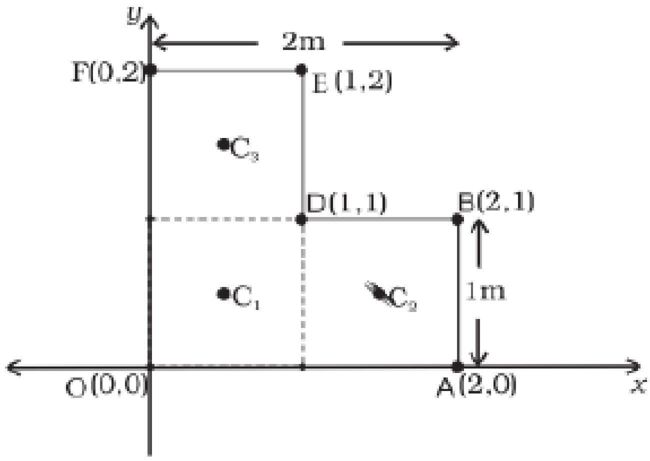
1. Three particles of equal masses are placed at co-ordinates (1, 1), (2, 2) and (4, 4) respectively. The position co-ordinate of COM of system of three particles is [NCERT Pg. 146]

(1) (0, 0) (2) $\left(\frac{2}{7}, \frac{7}{2}\right)$ (3) $\left(\frac{7}{3}, \frac{7}{3}\right)$ (4) (2, 2)

2. Consider a system of two identical particles. One of the particles is at rest and the other has an acceleration a. The centre of mass has an acceleration [NCERT Pg. 146)

(1) Zero (2) $\frac{1}{2}$a (3) a (4) 2 a

3. A thin uniform flat plate is in shape of *L* as shown. The mass of lamina is 6 kg. The position of centre of mass from point O [NCERT Pg. 147]



(1) $\left(\frac{5}{3}m, \frac{5}{3}m\right)$ (2) $\left(\frac{2}{3}m, \frac{5}{3}m\right)$ (3) $\left(\frac{1}{6}m, \frac{2}{6}m\right)$ (4) $\left(\frac{5}{6}m, \frac{5}{6}m\right)$

4. Which relation regarding product of two vectors is incorrect ? [NCERT Pg. 151]

(1) $\vec{a} \times \vec{a} = 0$ (2) $a.(\vec{b}+\vec{c}) = (\vec{a}.\vec{b}) + (\vec{a}.\vec{c})$

(3) $\vec{a} \times \vec{b} = -(-\vec{a}) \times (-\vec{b})$ (4) $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$

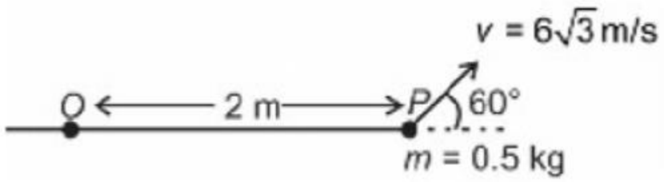
5. The vector product of given two vectors $\vec{A} = 3\hat{i} - 4\hat{j} + 5k$ and $\vec{B} = 2\hat{i} + \hat{j} + k$ is [NCERT Pg. 152]

(1) $-9\hat{i} + 13\hat{j} + 11k$ (2) $-9\hat{i} - 13\hat{j} + 11k$ (3) $-9\hat{i} + 7\hat{j} + 11k$ (4) $-9\hat{i} + 7\hat{j} - 11k$

6. The force acting on a particle is $(\hat{i} + 2\hat{j} + 3k)$. Find the torque of this force about origin if position vector of force is $(7\hat{i} + 3\hat{j} + 5k)$ m. [NCERT Pg. 157]

(1) $\hat{i} + 16\hat{j} - 11k$ (2) $-\hat{i} - 16\hat{j} + 11k$ (3) $\hat{i} + 16\hat{j} + 11k$ (4) $-\hat{i} + 9\hat{j} + 11k$

7. The angular momentum of a particle of mass 0.5 kg about point O at the instant as shown in the figure, is



(1) $6 kgm^2s^{-1}$ (2) $9 kgm^2s^{-1}$ (3) $18 kgm^2s^{-1}$ (4) $9\sqrt{3}\ kgm^2s^{-1}$

8. Which of the following statement is incorrect? [NCERT Pg. 158]

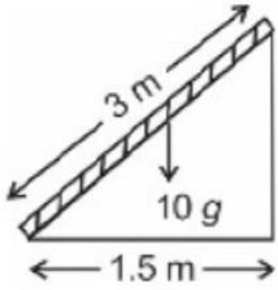
(1) Moment of couple is independent of point about which moment is taken.

(2) For translational equilibrium of a body vector sum of all the forces on it must be zero

(3) A body may be in translational equilibrium but may not be in rotational equilibrium simultaneously

(4) Rotational equilibrium depends on location of origin about which torques are taken

9. A 3 m long ladder weighing 10 kg leans on a frictionless wall. Its feet rest on floor 1.5 m from wall as shown. What is reaction force of the wall? [NCERT Pg. 162]

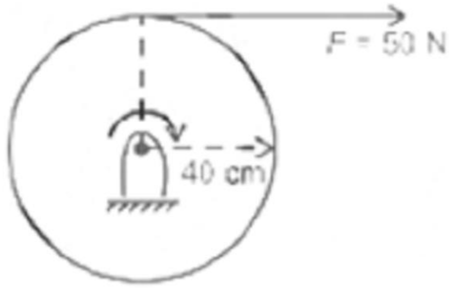


(1) $\frac{50}{\sqrt{3}}$ N (2) $50\sqrt{3}$N (3) $100\sqrt{3}$N (4) 120 N

10. Which of the following statement is incorrect? [NCERT Pg, 163]
(1) Moment of inertia depends on distribution of mass about rotational axis
(2) Moment of inertia depends on orientation and position of axis of rotation
(3) Moment of inertia changes when angular velocity of body changes
(4) Flywheel resists sudden increase or decrease of speed of vehicle

11. A ring has mass of 6 kg and radius of 2 m. What is moment of inertia of this ring about a tangent to the Circle of ring in its plane? [NCERT Pg. 166]
(1) 24 kg m^2 (2) 12kgm^2 (3) 30 kg m^2 (4) 36 kg m^2

12. A cord of negligible mass is wound round the rim of flywheel disc with mass of 15 kg and radius of 40 cm. A steady pull of 50 N is applied to cord as shown. The wheel is mounted on horizontal axis. What is angular acceleration of wheel?



(1) 10.33 rad s^{-2} (2) 16.66 rad s^{-2} (3) 20.66 rad s^{-2} (4) 4.99 rad s^{-2}

13. A cord of negligible mass is wrapped around a solid cylinder of a mass 20 kg and radius 20 cm. A steady pull of 25 N is applied on cord tangentially. The cylinder is mounted on horizontal axis with frictionless bearings. What is kinetic energy of wheel when 2 m cord is unwound? [NCERT Pg. 171]
(1) 50 J (2) 100J (3) 150J (4) 90 J

14. Four bodies; a ring, a solid cylinder, a hollow sphere and a solid sphere of same mass are allowed to roll down a rough inclined plane without slipping from same level- The body with greatest rotational kinetic energy at bottom is [NCERT Pg. 178]
(1) Ring (2) Solid cylinder (3) Hollow sphere (4) Solid sphere

15. A car weighs 1800 kg. The distance between its front axle and back axle is 1.8 m. Its centre of gravity is 1.05 m behind front axle. The force exerted by level ground on front wheels is (g = 10 ms^{-2}) [NCERT Pg. 178]
(1) 7500 N (2) 6500 N (3) 9500 N (4) 1800 N

16. A ring (circular) of radius 2 m has mass of 100 kg. It rolls purely along horizontal floor so that its COM has speed 20 cm s^{-1}. The work required to stop it is [NCERT Pg. 179]
(1) 2J (2) 3 J (3) 4J (4) 8J

17. To maintain a rotor at a uniform angular speed of 200 rad sH an engine needs to transmit a torque of 125 Nm. What is power required by the engine? [NCERT Pg. 179]
(1) 15 kW (2) 20 kW (3) 25 kW (4) 50 kW

18. A bullet of mass 10 gram is fixed with speed of 500 m s^{-1} into a door and gets embedded exactly at centre of door. The door is 1 m wide and weighs 12 kg. Door is hinged along one side and rotates about vertical axis without friction. The angular speed of door just after bullet embeds into it is [NCERT Pg. 180]
(1) 0.35 rad s^{-1} (2) 0.625 rad s^{-1}
(3) 0.255 rad s^{-1} (4) 0.935 rad s^{-1}

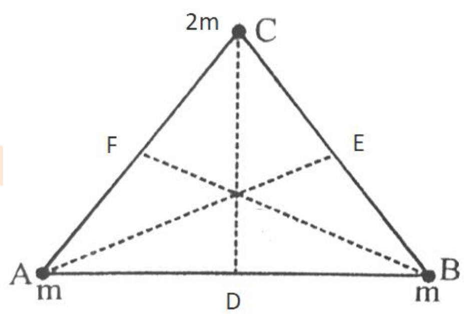
19. A solid disc of radius 10 cm are placed on a horizontal table (rough) with initial angular speed equal to 10 % rad s^{-1}. if coefficient of kinetic friction between disc and table is 0.2 then time taken by the disc to start pure rolling will be [NCERT Pg. 181]
(1) $\frac{\pi}{2}$s (2) $\frac{\pi}{3}$s (3) $\frac{\pi}{6}$s (4) $\frac{\pi}{4}$s

20. A child stands at centre of turntable with his two arms outstretched- The turntable is set rotating with angular speed of 40 rad s^{-1}. What will be angular speed of child if he folds his hands back reducing moment of inertia to $\frac{2}{5}$ times the initial value (ignore friction?) [NCERT Pg. 180]
(1) 50 rad s^{-1} (2) 75 rad s^{-1} (3) 100 rad s^{-1} (4) 150 rad s^{-1}

NCERT BASED PRACTICE QUESTIONS

1. Two particles of mass m_1 and m_2 are at distances r_1 and r_2 from the centre of mass then $\frac{r_1}{r_2}$ is
(a) $\frac{m_1}{m_2}$ (b) $\frac{m_2}{m_1}$ (c) $\frac{m_1+m_2}{m_1}$ (d) $\frac{m_1+m_2}{m_2}$

2. Two balls each of mass m are placed on two vertices of an equilateral triangle. A ball of 2m is situated at third vertex then the centre of mass of the system is at mid point of



(a) bisector of AE (b) bisector of BF
(c) bisector CD (d) AB

3. The identical spheres A,B and C each of radius R are placed touching each other on a horizontal table. Then centre of mass of the system relative to the centre of sphere A is
(a) $\frac{1}{3}\left(\overline{AB}+\overline{AC}\right)$ (b) $\frac{1}{2}\left(\overline{AB}+\overline{AC}\right)$ (c) $\overline{AB}+\overline{AC}$ (d) none of these

4. Which of the following statement is most appropriately correct?
(a) centre of mass of the body must lie inside the body
(b) centre of mass may lie inside the body

(c) centre of mass must lie outside the body
(d) centre of mass may lie inside or out side the body

5. If two particles of mass m_1 and m_2 are placed at a distance r_0 from each other then distance of particle of mass m_2 from centre of mass is

(a) $\frac{m_2}{m_1+m_2}r_0$ (b) $\frac{m_1}{m_1+m_2}r_0$ (c) $\frac{m_1}{m_2}r_0$ (d) $\frac{m_2}{m_1}r_0$

6. Two particle of same mass are placed at $(x_1,0)$ $(x_2,)$ then x co-ordinate of centre of mass is

(a) $\frac{x_1+x_2}{2}$ (b) $\frac{x_1-x_2}{2}$ (c) x_1 (d) x_2

7. If a body is projected at same angle from horizontal – a horizontal range R. If the body explodes in mid air then The centre of mass of the exploded particle will fall a distance a

(a) $\frac{R}{4}$ (b) $\frac{R}{2}$ (c) $\frac{3R}{4}$ (d) R

8. The motor of an engine is rotating about its axis with an angular velocity of 120 r. P.m. If comes to rest in 105 after being switched off. Number of revolution made by the motor before coming to rest is

(a) 5 (b) 15 (c) 10 (d) 20

9. Torque due to a force is due to

(a) transverse component (b) longitudinal component
(c) by all the component (d) none of these

10. A body is called in rotational equilibrium when

(a) clockwise moment > anticlockwise moment
(b) clockwise moment < anticlockwise moment
(c) clockwise moment = anticlockwise moment
(d) none of these

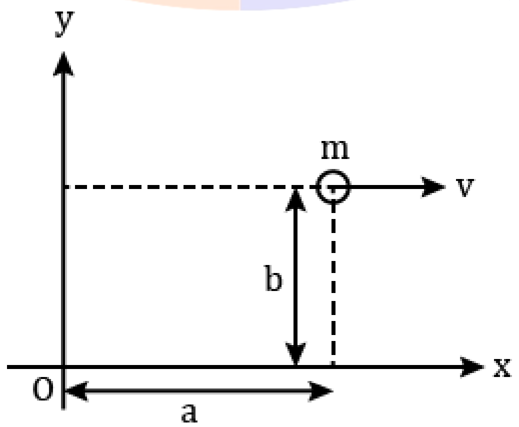
11. If two force of magnitude F are acting on the opposite side of a rod of length l then torque of the forces about mid point of rod is

(a) $\frac{Fl}{2}$ (b) *Fl* (c) *F2l* (d) *F 4l*

12. Torque of a force is given by a formula

(a) $\bar{\tau}=\bar{r}\times\bar{F}$ (b) $\tau=\bar{r}.\bar{F}$ (c) $\bar{\tau}=\bar{r}+\bar{F}$ (d) none of these

13. A particle of mass m is moving with a constant velocity v parallel to the x− axis as shown in the figure. Its angular momentum (magnitude) about origin O is



a) mvb   b) mva   c) $mv\sqrt{a^2+b^2}$ d) mv(a+b)

14. Which of the following relation is correct?
(a) angular momentum = 2 (mass × areal velocity)
(b) angular momentum = mass × areal velocity
(c) angular momentum = $\frac{1}{2}$ (mass × areal velocity)
(d) none of these

15. A rigid body is said in mechanical equilibrium when linear momentum (P) and angular momentum (L) are
(a) P = constant; L = constant   (b) P ≠ constant, L = constant
(c) P = constant; L ≠ constant   (d) P ≠ constant, L ≠ constant

16. Which of the following statement is correct about centre of mass and centre of gravity?
(a) both are at same point   (b) both may be at same point
(c) both must be at different point   (d) none of these

17. If radius of earth decreases and its mass remain constant then time period of day time
(a) increases   (b) decreases
(c) remains constant   (d) con not be said

18. Moment of inertial of a ring of mass m and radiurs r about its diameter is
(a) mr^2 (b) $\frac{1}{2}mr^2$ (c) $\frac{1}{4}mr^2$ (d) $2\ mr^2$

19. Moment of inertial of a disc of mass m and radius r about its diameter is
(a) mr^2 (b) $\frac{1}{2}mr^2$ (c) $\frac{1}{4}mr^2$ (d) $2\ mr^2$

20. Moment of inertia of a solid sphere of mass m and radius r about t diameter is
(a) $\frac{2}{3}mr^2$ (b) $\frac{2}{5}mr^2$ (c) $\frac{1}{2}mr^2$ (d) mr^2

21. Moment of inertia of a body depends on
(a) axis of rotation   (b) distribution of mass about rotation axis
(c) both a and b   (d) none of these

22. Moment of inertia of a ring about an axis tangent to the ring and in the plane of the ring is
(a) $\frac{3}{2}mr^2$ (b) mr^2 (c) $\frac{1}{2}mr^2$ (d) $\frac{3}{4}mr^2$

23. A body of mass m and radius of gyration K and radius R is purely rolling with centre of mass velocity υ then kinetic energy of the body is
(a) $\frac{1}{2}m\upsilon^2$ (b) $\frac{1}{2}m\upsilon^2\left(1+\frac{k^2}{R^2}\right)$
(c) $\frac{1}{2}m\upsilon^2\left(1+\frac{R^2}{K^2}\right)$ (d) None

24. A body is rolling down on an inclined plane of angle θ radius of the body is R and radius of gyration K. Then acceleration of the body is
(a) $I\sin\theta$ (b) $\frac{I\sin\theta}{1+\frac{R^2}{k^2}}$ (c) $\frac{I\sin\theta}{1+\frac{K^2}{R^2}}$ (d) $\frac{I\cos\theta}{1+\frac{R^2}{k^2}}$

25. A ring, a disc and a sphere all of the same redius and mass roll down on an inclined plane of inclination θ from a height h Then which of the following will reach the bottom first
(a) ring (b) disc
(c) sphere (d) all will reach simultaneously

26. A sphere is rolling on a level surface find the ratio of the kinetic energy due to the translational motion to the total energy of the sphere?
(a) $\frac{2}{7}$ (b) $\frac{7}{5}$ (c) $\frac{5}{7}$ (d) $\frac{5}{9}$

27. A thin circular ring of mass M and radius R is rotating about its axis with a constant angular velocity w. Four objects each of mass m are kept gently to the opposite ends of two perpendicular diameters of the ring angular velocity of the ring will be
(a) $\frac{M}{M+4m}w$ (b) $\frac{M}{4m}w$ (c) $\frac{M+4m}{M}w$ (d) $\frac{m}{4M}w$

28. A particle performing uniform circular motion has angular momentum L. What will be its angular momentum if its angular frequency is halved and kinetic energy is doubled?
(a) 3L (b) 2L (c) 4 L (d) 5 L

29. What will be the duration of the day ,it the earth suddenly shrinks to $\frac{1}{64}$ of its original volume mass remaining unchanged?
(a) 1.5 h (b) 3h (c) 6 h (d) 8h

30. Form a uniform disk of radius R, a circular hole of radius R/2 is cut out. The centre of the hole is at R/2 from the centre of the original disc. Ten distance of centre of gravity of the remaining body from centre is
(a) $\frac{R}{8}$ (b) $\frac{R}{2}$ (c) $\frac{R}{6}$ (d) $\frac{R}{12}$

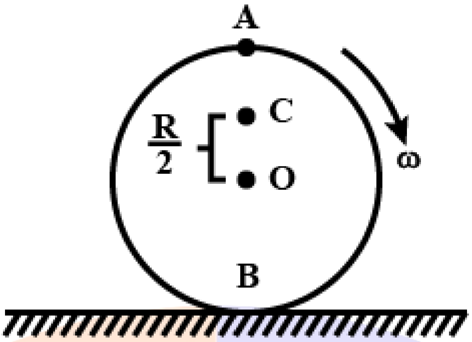
31. A rope of negligible mass is wound round a hollow cylinder of mass 3kg and radius 40cm. What is the angular acceleration of cylinder if the rope is pulled with a force of 30N
(a) 20 m/s^2 (b) 15 m/s^2 (c) 25 m/s^2 (d) 10 m/s^2

32. A hoop of radius 2 m weighs 100kg. If rolls along a horizontal floor so that its centre of mass has a speed of 20cm/s.How much work has to be done to stop it
(a) 2 J (b) 3 J (c) 4 J (d) 6 J

33. A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5g are put one on top of the other at the 12.0 cm mark. The stick is found to be balanced at 45.0cm. Mass of the metre stick is
(a) 56 gm (b) 66 gm (c) 64 gm (d) 76 gm

34. If a body of radius R and radius of gyration K is rolling on an inclined plane of height h. Then translational velocity of the body at the bottom of plane is
(a) $\frac{Ih}{1+\frac{k^2}{R^2}}$ (b) $\frac{Ih}{1+\frac{R^2}{k^2}}$ (c) $\frac{2Ih}{1+\frac{k^2}{R^2}}$ (d) none of these

35. A disc rotating about its axis with angular speed W_0 is placed lightly on a perfectly frictionless table. The radius of the disc is R $\upsilon_A, \upsilon_B, \upsilon_c$ is speed of point A,B & C then



(a) $\upsilon_A < \upsilon_B < \upsilon_c$ (b) $\upsilon_A > \upsilon_C > \upsilon_B$
(c) $\upsilon_A < \upsilon_B > \upsilon_c$ (d) $\upsilon_C > \upsilon_B > \upsilon_A$

36. A solid cylinder rolls up an inclined plane of angle of inclination 30^0. At the bottom of the inclined plane the centre of mass of the cylinder has a speed of 5m/s. How long will it take to return to the bottom
(a) 2 s (b) 3 s (c) 4 s (d) 3.5 s

37. A solid cylinder of mass 20kg rotates about its axis with angular speed 100 red/s. The radius of the cylinder is 0.25 m . The kinetic energy associated with the rotation of the cylinder is
(a) 6250 J (b) 3100 J (c) 3125 J (d) 3000 J

38. A car weighs 1800 kg. The distance between its front and back axles is 1.8m. Its centre of gravity is 1.05 m behind the front axles. Force exerted by the level ground on front wheel is
(a) 5145 N (b) 3675 N (c) 10290 N (d) 7350 N

39. The moment of inertia of a sphere about a tangent to the sphere is.
(a) $\frac{7}{5}MR^2$ (b) $\frac{2}{5}MR^2$ (c) $\frac{2}{3}MR^2$ (d) $\frac{1}{2}MR^2$

40. Four point masses each of the mass M are placed at the corners of a squre ABCD of side L. The moment of inertia of this system about an axis passing through A and parallel to BD is
(a) 3 ML^2 (b) ML^2 (c) 2 ML^2 (d) $\sqrt{3}ML^2$

TOPIC WISE PRACTICE QUESTIONS

Topic 1: Centre of Mass, Angular Velocity & Acceleration

1. Centre of mass of the earth and the moon system lies
(1) closer to the earth (2) closer to the moon
(3) at the mid-point of line joining the earth and the moon (4) cannot be predicted

2. The centre of mass of two particles lies on the line
(1) joining the particles (2) perpendicular to the line joining the particles
(3) at any angle to this line (4) None of these

3. A mass is revolving in a circle which is in the plane of paper. The direction of angular acceleration is
(1) upward the radius (2) towards the radius (3) tangential (4) at right angle to angular velocity

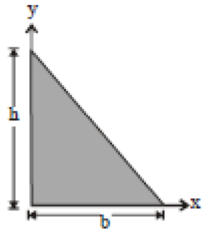
4. In rotatory motion, linear velocities of all the particles of the body are
(1) same (2) different (3) zero (4) cannot say

5. Two bodies A and B have masses M and m respectively where M > m and they are at a distance d apart. Equal force is applied to them so that they approach each other. The position where they hit each other is
(1) nearer to B (2) nearer to A (3) at equal distance from A and B (4) cannot be determined

6. A pulley fixed to the ceiling carries a string with blocks of mass m and 3 m attached to its ends. The masses of string and pulley are negligible. When the system is released, its centre of mass moves with what acceleration?
(1) 0 (2) – g/4 (3) g/2 (4) – g/2

7. The wheel of a car is rotating at the rate of 1200 revolutions per minute. On pressing the accelerator for 10 seconds. It starts rotating at 4500 revolutions per minute. The angular acceleration of the wheel is
(1) 30 radians/second^2 (2) 1880 degree/second^2 (3) 40 radians/second^2 (4) 1980 degree/second^2

8. The centre of mass of triangle shown in figure has coordinates



(1) $x=\frac{h}{2}, y=\frac{b}{2}$ (2) $x=\frac{b}{2}, y=\frac{h}{2}$ (3) $x=\frac{b}{3}, y=\frac{h}{3}$ (4) $x=\frac{h}{3}, y=\frac{b}{3}$

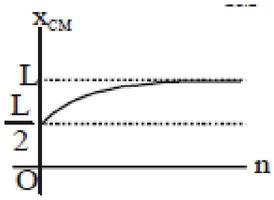
9. When a ceiling fan is switched off, its angular velocity falls to half while it makes 36 rotations. How many more rotations will it make before coming to rest?
(1) 24 (2) 36 (3) 18 (4) 12

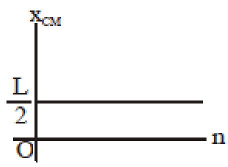
10. Two identical particles are located at $\vec{x}$ and $\vec{y}$ with reference to the origin of three-dimensional co-ordinate system. The position vector of centre of mass of the system is given by
(1) $\vec{x}-\vec{y}$ (2) $\frac{\vec{x}+\vec{y}}{2}$ (3) $(\vec{x}-\vec{y})$ (4) $\frac{\vec{x}-\vec{y}}{2}$

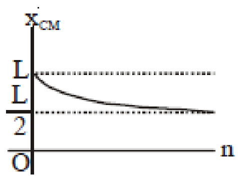
11. In a bicycle, the radius of rear wheel is twice the radius of front wheel. It r_f and r_r are the radii and v_f and v_r are the speeds of topmost points of wheels then
(1) $v_r = 2v_f$ (2) $v_f = 2v_r$ (3) $v_f = v_r$ (4) $v_f = 4v_r$

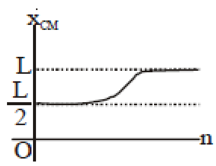
12. In carbon monoxide molecule, the carbon and the oxygen atoms are separated by a distance 1.12×10^{-10} m. The distance of the centre of mass, from the carbon atom is
(1) 0.64×10^{-10} m (2) 0.56×10^{-10} m (3) 0.51×10^{-10} m (4) 0.48×10^{-10} m

13. A thin rod of length 'L' is lying along the x-axis with its ends at $x = 0$ and $x = L$. Its linear density (mass/length) varies with x as $k\left(\frac{x}{L}\right)^n$, where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against 'n', which of the following graphs best approximates the dependence of x_{CM} on n?

(1)


(2)


(3)


(4)


14. Two particles A and B, initially at rest, moves towards each other under a mutual force of attraction. At the instant when the speed of A is v and the speed of B is 2 v, the speed of centre of mass is
(1) 2v (2) v (3) 1.5 v (4) zero

15. A stick is thrown in the air and lands on the ground at some distance from the thrower. The centre of mass of the stick will move along a parabolic path
(1) in all cases

(2) only if the stick is uniform
(3) only if the stick has linear motion but no rotational motion
(4) only if the stick has a shape such that its centre of mass is located at some point on it and not outside it

16. A small disc of radius 2 cm is cut from a disc of radius 6 cm. If the distance between their centres is 3.2 cm, what is the shift in the centre of mass of the disc?
(1) 0.4 cm (2) 2.4 cm (3) 1.8 cm (4) 1.2 cm

Topic 2: Torque, Couple and Angular Momentum

17. Rotational analogue of force in linear motion is
(1) weight (2) angular momentum (3) moment of inertia (4) torque

18. A disc is given a linear velocity on a rough horizontal surface then its angular momentum is
(1) conserved about COM only (2) conserved about the point of contact only
(3) conserved about all the points (4) not conserved about any point.

19. A particle of mass m is moving in a plane along a circular path of radius r. Its angular momentum about the axis of rotation is L. The centripetal force acting on the particle is
(1) L^2/mr (2) $L^2 m/r$ (3) $L^2/m r^3$ (4) $L^2/m r^2$

20. A stone of mass m tied to a string of length l is rotating along a circular path with constant speed v. The torque on the stone is
(1) mlv (2) mv/l (3) mv2/l (4) zero

21. When a mass is rotating in a plane about a fixed point, its angular momentum is directed along the
(1) radius of orbit (2) tangent to the orbit
(3) line parallel to plane of rotation (4) line perpendicular to plane of rotation

22. A particle of mass 0.2 kg is moving in a circle of radius 1 m with $f = (2/\pi)\ sec^{-1}$, then its angular momentum is :
(1) 0.8 kg-m²/s (2) 2 kg-m²/s (3) 8 kg-m²/s (4) 16 kg-m²/s

23. A man standing on a rotating table is holding two masses at arm's length. Without moving his arms, he drops the two masses. His angular speed will
(1) increase (2) decrease (3) become zero (4) remain constant

24. A wheel having moment of inertia 2 kg-m² about its vertical axis, rotates at the rate of 60 rpm about this axis, The torque which can stop the wheel's rotation in one minute would be
(1) $\frac{\pi}{18}$ Nm (2) $\frac{2\pi}{15}$ Nm (3) $\frac{\pi}{12}$ Nm (4) $\frac{\pi}{15}$ Nm

25. A smooth sphere A is moving on a frictionless horizontal plane with angular speed ω and centre of mass velocity v. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B, respectively. Then
(1) $\omega_A < \omega_B$ (2) $\omega_A = \omega_B$ (3) $\omega_A = \omega$ (4) $\omega_B = \omega$

26. A particle of mass *m* is moving in a circle of radius *r*. The centripetal acceleration (a_c) of the particle varies with the time according to the relation, $a_c = Kt^2$, where K is a positive constant and *t* is the time. The magnitude of the time rate of change of angular momentum of the particle about the centre of the circle is
(1) mKr (2) $\sqrt{m^2Kr^3}$ (3) $\sqrt{mKr}$ (4) mKr^2

27. A particle of mass 2 kg is moving such that at time *t*, its position, in meter, is given by $\vec{r}(t) = 5\hat{i} - 2t^2\hat{j}$. The angular momentum of the particle at *t* = 2*s* about the origin in kg $m^{-2}\ s^{-1}$ is :
(1) $-80\hat{k}$ (2) $(10\hat{i} - 16\hat{j})$ (3) $-40\hat{k}$ (4) $40\hat{k}$

28. A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0 m wide and weighs 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction. The angular speed of the door just after the bullet embeds into it will be :
(1) 6.25 rad/sec (2) 0.625 rad/sec (3) 3.35 rad/sec (4) 0.335 rad/sec

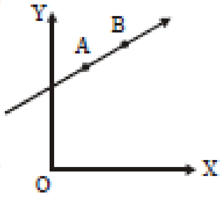
29. A stone of mass *m*, tied to the end of a string, is whirled around in a circle on a horizontal frictionless table. The length of the string is reduced gradually keeping the angular momentum of the stone about the centre of the circle constant. Then, the tension in the string is given by $T = Ar^n$, where A is a constant, r is the instantaneous radius of the circle. The value of n is equal to

(1) – 1 (2) – 2 (3) – 4 (4) – 3

30. A flywheel rotates about an axis. Due to friction at the axis, it experiences an angular retardation proportional to its angular velocity. If its angular velocity falls to half while it makes n rotations, how many more rotations will it make before coming to rest?

(1) 2n (2) n (3) n/2 (4) n/3

31. A particle of mass m moves in the XY plane with a velocity v along the straight line AB. If the angular momentum of the particle with respect to origin O is L_A when it is at A and L_B when it is at B, then



(1) $L_A = L_B$ (2) $L_A > L_B$ (3) $L_A < L_B$

d) the relationship between L_A and L_B depends upon the slope of the line AB

32. The angular momentum of a particle relative to a point O varies with time as $\boldsymbol{J} = \boldsymbol{a} + \boldsymbol{b}t^2$, where $\vec{a}$ and $\vec{b}$ are constant vectors, with $\vec{a}$ perpendicular $\vec{b}$. The moment of force (τ) relative to the point O acting on the particle when the angle between the vectors $\vec{\tau}$ and $\boldsymbol{J}$ equal to 45°:

(1) $\sqrt{\frac{a}{b}}$ (2) $2\sqrt{\frac{a}{b}}$ (3) $2b\sqrt{\frac{a}{b}}$ (4) $2a\sqrt{\frac{b}{a}}$

Topic 3: Moment of Inertia & Radius of Gyration

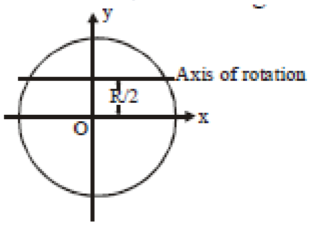
33. Of the two eggs which have identical sizes, shapes and weights, one is raw, and other is half boiled. The ratio between the moment of inertia of the raw to the half boiled egg about central axis is

(1) one (2) greater than one (3) less than one (4) not comparable

34. A billiard ball of mass m and radius r, when hit in a horizontal direction by a cue at a height h above its centre, acquired a linear velocity v_0. The angular velocity ω_0 acquired by the ball is

(1) $\frac{5v_0r^2}{2h}$ (2) $\frac{2v_0r^2}{5h}$ (3) $\frac{2v_0h}{5r^2}$ (4) $\frac{5v_0h}{2r^2}$

35. M.I of a circular loop of radius R about the axis in figure is



(1) MR^2 (2) (3/4) MR^2 (3) $MR^2/2$ (4) $2MR^2$

36. Consider a uniform square plate of side 'a' and mass 'M'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

(1) $\frac{5}{6}Ma^2$ (2) $\frac{1}{12}Ma^2$ (3) $\frac{7}{12}Ma^2$ (4) $\frac{2}{3}Ma^2$

37. The angular velocity of a body changes from ω_1 to ω_2 without applying a torque but by changing the moment of inertia about its axis of rotation. The ratio of its corresponding radii of gyration is

(1) $\omega_1 : \omega_2$ (2) $\sqrt{\omega_1} : \sqrt{\omega_2}$ (3) $\omega_2 : \omega_1$ (4) $\sqrt{\omega_2} : \sqrt{\omega_1}$

38. A circular disc A of radius r is made from an iron plate of thickness t and another circular disc B of radius 4r is made from an iron plate of thickness t/4. The relation between the moments of inertia I_A and I_B is

(1) $I_A > I_B$ (2) $I_A = I_B$ (3) $I_A < I_B$ (4) depends on the actual value of t and r

39. If the moment of inertia of a disc about an axis tangential and parallel to its surface be I, then what will be the moment of inertia about the axis tangential but perpendicular to the surface?

(1) $\frac{6}{5}I$ (2) $\frac{3}{4}I$ (3) $\frac{3}{2}I$ (4) $\frac{5}{4}I$

40. A circular turn table has a block of ice placed at its centre. The system rotates with an angular speed ω about an axis passing through the centre of the table. If the ice melts on its own without any evaporation, the speed of rotation of the system

(1) becomes zero (2) remains constant at the same value ω

(3) increases to a value greater than ω (4) decreases to a value less than w

41. The moment of inertia of a disc of mass M and radius R about an axis, which is tangential to the circumference of the disc and parallel to its diameter, is

(1) $\frac{3}{2}MR^2$ (2) $\frac{2}{3}MR^2$ (3) $\frac{5}{4}MR^2$ (4) $\frac{4}{5}MR^2$

42. Consider a thin uniform square sheet made of a rigid material. If its side is '*a*' mass *m* and moment of inertia I about one of its diagonals, then :

(1) $I > \frac{ma^2}{12}$ (2) $\frac{ma^2}{24} < I < \frac{ma^2}{12}$ (3) $I = \frac{ma^2}{24}$ (4) $I = \frac{ma^2}{12}$

43. Moment of inertia of a uniform circular disc about a diameter is I. Its moment of inertia about an axis perpendicular to its plane and passing through a point on its rim will be

(1) 5 I (2) 3 I (3) 6 I (4) 4 I

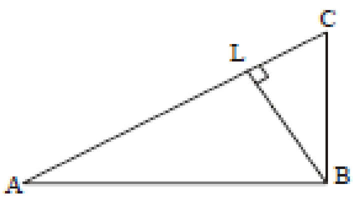
44. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius 4R is made from an iron plate of thickness t/4. Then the relation between the moment of inertia I_X and I_Y is

(1) $I_Y = 32\, I_X$ (2) $I_Y = 16\, I_X$ (3) $I_Y = I_X$ (4) $I_Y = 64\, I_X$

45. Point masses 1, 2, 3 and 4 kg are lying at the points (0, 0, 0), (2, 0, 0), (0, 3, 0) and (–2, –2, 0) respectively. The moment of inertia of this system about X-axis will be

(1) 43 kg m^2 (2) 34 kg m^2 (3) 27 kg m^2 (4) 72 kg m^2

46. About which axis moment of inertia in the given triangular lamina is maximum



(1) AB (2) BC (3) AC (4) BL

47. Initial angular velocity of a circular disc of mass *M* is ω_1. Then two small spheres of mass *m* are attached gently to diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?

(1) $\left(\frac{M+m}{M}\right)\omega_1$ (2) $\left(\frac{M+m}{m}\right)\omega_1$ (3) $\left(\frac{M}{M+4m}\right)\omega_1$ (4) $\left(\frac{M}{M+2m}\right)\omega_1$

Topic 4: Rotational Kinetic Energy and Rolling Motion

48. A solid sphere, disc and solid cylinder all of the same mass and made of the same material are allowed to roll down (from rest) on an inclined plane, then

(1) solid sphere reaches the bottom first (2) solid sphere reaches the bottom last

(3) disc will reach the bottom first (4) all reach the bottom at the same time

49. A rod PQ of length L revolves in a horizontal plane about the axis YY´. The angular velocity of the rod is ω. If A is the area of cross-section of the rod and ρ be its density, its rotational kinetic energy is

(1) $\frac{1}{3}AL^3\rho\omega^2$ (2) $\frac{1}{2}AL^3\rho\omega^2$ (3) $\frac{1}{24}AL^3\rho\omega^2$ (4) $\frac{1}{18}AL^3\rho\omega^2$

50. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

(1) $\frac{r\omega_0}{4}$ (2) $\frac{r\omega_0}{3}$ (3) $\frac{r\omega_0}{2}$ (4) $r\omega_0$

51. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is K. If radius of the ball be R, then the fraction of total energy associated with its rotational energy will be

(1) $\frac{K^2}{R^2}$ (2) $\frac{K^2}{K^2+R^2}$ (3) $\frac{R^2}{K^2+R^2}$ (4) $\frac{K^2+R^2}{R^2}$

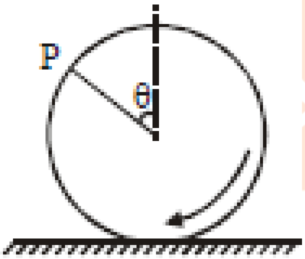
52. A solid sphere of mass 2 kg rolls on a smooth horizontal surface at 10 m/s. It then rolls up a smooth inclined plane of inclination 30° with the horizontal. The height attained by the sphere before it stops is

(1) 700 cm (2) 701 cm (3) 7.1 m (4) None of these

53. A sphere rolls down on an inclined plane of inclination θ. What is the acceleration as the sphere reaches the bottom?

(1) $\frac{5}{7}g\sin\theta$ (2) $\frac{3}{5}g\sin\theta$ (3) $\frac{2}{7}g\sin\theta$ (4) $\frac{2}{5}g\sin\theta$

54. A wheel is rolling straight on ground without slipping. If the axis of the wheel has speed v, the instantenous velocity of a point P on the rim, defined by angle θ, relative to the ground will be

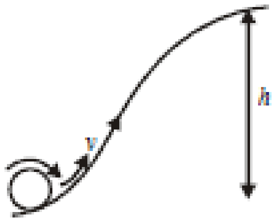


(1) $v\cos\left(\frac{1}{2}\theta\right)$ (2) $2v\cos\left(\frac{1}{2}\theta\right)$ (3) $v(1+\sin\theta)$ (4) $v(1+\cos\theta)$

55. A body having moment of inertia about its axis of rotation equal to 3 kg-m^2 is rotating with angular velocity equal to 3 rad/s. Kinetic energy of this rotating body is the same as that of a body of mass 27 kg moving with a speed of

(1) 1.0 m/s (2) 0.5 m/s (3) 1.5 m/s (4) 2.0 m/s

57. A solid sphere is rolling on a surface as shown in figure, with a translational velocity *v* ms^{-1}. If it is to climb the inclined surface continuing to roll without slipping, then minimum velocity for this to happen is



(1) $\sqrt{2gh}$ (2) $\sqrt{\frac{7}{5}gh}$ (3) $\sqrt{\frac{7}{2}gh}$ (4) $\sqrt{\frac{10}{7}gh}$

58. The moment of inertia of a body about a given axis is 1.2 kg m^2. Initially, the body is at rest. In order to produce a rotational kinetic energy of 1500 joule, an angular acceleration of 25 radian/sec^2 must be applied about that axis for a duration of

(1) 4 seconds (2) 2 seconds (3) 8 seconds (4) 10 seconds

59. A uniform solid cylindrical roller of mass 'm' is being pulled on a horizontal surface with force F parallel to the surface and applied at its centre. If the acceleration of the cylinder is 'a' and it is rolling without slipping then the value of 'F' is:

(1) ma (2) $\frac{5}{3}$ma (3) $\frac{3}{2}$ma (4) 2ma

60. A body rolls down an inclined plane. If its K.E. of rotational motion is 40% of its K.E. of translational, then the body is a
(1) cylinder (2) ring (3) solid disc (4) solid sphere

NEET PREVIOUS YEARS QUESTIONS

1. Three objects, A : (a solid sphere), B : (a thin circular disk) and C : (a circular ring), each have the same mass M and radius R. They all spin with the same angular speed ω about their own symmetry axes. The amounts of work (W) required to bring them to rest, would satisfy the relation **[2018]**
(1) $W_C > W_B > W_A$ (2) $W_A > W_B > W_C$ (3) $W_A > W_C > W_B$ (4) $W_B > W_A > W_C$

2. A solid sphere is in rolling motion. In rolling motion a body possesses translational kinetic energy (K_t) as well as rotational kinetic energy (K_r) simultaneously. The ratio $K_t : (K_t + K_r)$ for the sphere is **[2018]**
(1) 7 : 10 (2) 5 : 7 (3) 2 : 5 (4) 10 : 7

3. A rope is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N ? **[2017]**
(1) 0.25 rad/s^2 (2) 25 rad/s^2 (3) 5 m/s^2 (4) 25 m/s^2

4. Two discs of same moment of inertia rotating about their regular axis passing through centre and perpendicular to the plane of disc with angular velocities ω_1 and ω_2. They are brought into contact face to face coinciding the axis of rotation. The expression for loss of energy during this process is:- **[2017]**
(1) $\frac{1}{4}I(\omega_1-\omega_2)^2$ (2) $I(\omega_1-\omega_2)^2$ (3) $\frac{1}{8}(\omega_1-\omega_2)^2$ (4) $\frac{1}{2}(\omega_1+\omega_2)^2$

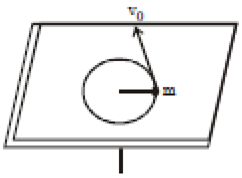
5. Which of the following statements are **correct**? **[2017]**
(1) Centre of mass of a body always coincides with the centre of gravity of the body
(2) Centre of mass of a body is the point at which the total gravitational torque on the body is zero
(3) A couple on a body produce both translational and rotation motion in a body
(4) Mechanical advantage greater than one means that small effort can be used to lift a large load
(1) (1) and (2) (2) (2) and (3) (3) (3) and (4) (4) (2) and (4)

6. From a disc of radius R and mass M, a circular hole of diameter R, whose rim passes through the centre is cut. What is the moment of inertia of the remaining part of the disc about a perpendicular axis, passing through the centre? **[2016]**
(1) 15 MR^2/32 (2) 13 MR^2/32 (3) 11 MR^2/32 (4) 9 MR^2/32

7. A uniform circular disc of radius 50 cm at rest is free to turn about an axis which is perpendicular to its plane and passes through its centre. It is subjected to a torque which produces a constant angular acceleration of 2.0 rad s^{-2}. Its net acceleration in ms^{-2} at the end of 2.0s is approximately: **[2016]**
(1) 8.0 (2) 7.0 (3) 6.0 (4) 3.0

8. A disk and a sphere of same radius but different masses roll off on two inclined planes of the same altitude and length. Which one of the two objects gets to the bottom of the plane first ? **[2016]**
(1) Disk (2) Sphere
(3) Both reach at the same time (4) Depends on their masses

9. A mass m moves in a circle on a smooth horizontal plane with velocity v_0 at a radius R_0. The mass is attached to string which passes through a smooth hole in the plane as shown.



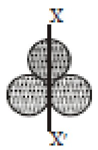
The tension in the string is increased gradually and finally m moves in a circle of radius $\frac{R_0}{2}$. The final value of the kinetic energy is **[2015]**

(1) $\frac{1}{4}mv_0^2$ (2) $2mv_0^2$ (3) $\frac{1}{2}mv_0^2$ (4) mv_0^2

10. A rod of weight W is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. The normal reaction on A is **[2015]**

(1) $\frac{Wd}{x}$ (2) $\frac{W(d-x)}{x}$ (3) $\frac{W(d-x)}{d}$ (4) $\frac{Wx}{d}$

11. Three identical spherical shells, each of mass m and radius r are placed as shown in figure. Consider an axis XX' which is touching to two shells and passing through diameter of third shell. Moment of inertia of the system consisting of these three spherical shells about XX' axis is **[2015]**

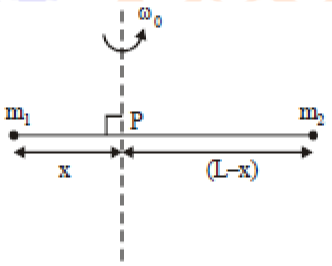


(1) $3mr^2$ (2) $\frac{16}{5}mr^2$ (3) $4mr^2$ (4) $\frac{11}{5}mr^2$

12. An automobile moves on a road with a speed of 54 km h^{-1}.The radius of its wheels is 0.45 m and the moment of inertia of the wheel about its axis of rotation is 3 kg m^2. If the vehicle is brought to rest in 15s, the magnitude of average torque transmitted by its brakes to the wheel is : **[2015]**

(1) 8.58 kg m^2 s^{-2} (2) 10.86 kg m^2 s^{-2} (3) 2.86 kg m^2 s^{-2} (4) 6.66 kg m^2 s^{-2}

13. Point masses m_1 and m_2 are placed at the opposite ends of a rigid rod of length L, and negligible mass. The rod is to be set rotating about an axis perpendicular to it. The position of point P on this rod through which the axis should pass so that the work required to set the rod rotating with angular velocity ω_0 is minimum, is given by **[2015]**



(1) $x = \frac{m_1}{m_2}L$ (2) $x = \frac{m_2}{m_1}L$ (3) $x = \frac{m_2 L}{m_1 + m_2}$ (4) $x = \frac{m_1 L}{m_1 + m_2}$

14. A force $\vec{F} = \alpha\hat{i} + 3\hat{j} + 6\hat{k}$ is acting at a point $\vec{r} = 2\hat{i} - 6\hat{j} - 12\hat{k}$. The value of α for which angular momentum about origin is conserved is : **[2015]**

(1) 2 (2) zero (3) 1 (4) –1

15. A solid cylinder of mass 50 kg and radius 0.5 m is free to rotate about the horizontal axis. A massless string is wound round the cylinder with one end attached to it and other hanging freely. Tension in the string required to produce an angular acceleration of 2 revolutions s^{-2} is : **[2014]**

(1) 25 N (2) 50 N (3) 78.5 N (4) 157 N

16. The ratio of the accelerations for a solid sphere (mass 'm' and radius 'R') rolling down an incline of angle 'θ' without slipping and slipping down the incline without rolling is : **[2014]**

(1) 5 : 7 (2) 2 : 3 (3) 2 : 5 (4) 7 : 5

17. A disc of radius 2m and mass 100 kg rolls on a horizontal floor. Its centre of mass has speed of 20 cm/s. How much work is needed to stop it ? **[NEET-2019]**

(1) 3J (2) 30 kJ (3) 2 J (4) 1 J

18. A solid cylinder of mass 2 kg and radius 4 cm is rotating about its axis at the rate of 3 rpm. The torque required to stop after 2p revolutions is : **[NEET-2019]**

(1) 2×10^{-6} N m (2) 2×10^{-3} N m (3) 12×10^{-4} N m (4) 2×10^{6} N m

19. Two particles A and B are moving in uniform circular motion in concentric circles of radius r_A and r_B

with speed v_A and v_B respectively. The time period of rotation is the same. The ratio of angular speed of A to that of B will be : **[NEET-2019]**

(1) $r_A : r_B$ (2) $v_A : v_B$ (3) $r_B : r_A$ (4) 1 : 1

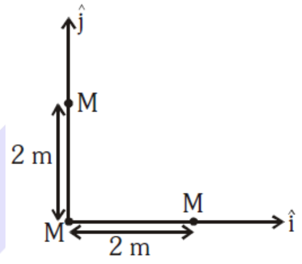
20. A solid cylinder of mass 2 kg and radius 50 cm rolls up an inclined plane of angle inclination 30°. The centre of mass of cylinder has speed of 4 m/s. The distance travelled by the cylinder on the incline surface will be : (Take g = 10 m/s^2) **[NEET – 2019 (ODISSA)]**

(1) 2.2 m (2) 1.6 m (3) 1.2 m (4) 2.4 m

21. The angular speed of the wheel of a vehicle is increased from 360 rpm to 1200 rpm in 14 second. Its angular acceleration is **NEET-2020(COVID-19)**

(1) 2π rad/s^2 (2) 28π rad/s^2 (3) 120π rad/s^2 (4) 1 rad/s^2

22. Three identical spheres, each of mass M, are placed at the corners of a right angle triangle with mutually perpendicular sides equal to 2 m (see figure). Taking the point of intersection of the two mutually perpendicular sides as the origin, find the position vector of centre of mass. **NEET-2020(COVID-19)**

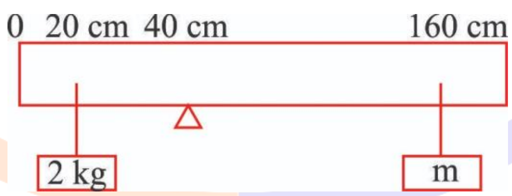


(1) $2(\hat{i}+\hat{j})$ (2) $(\hat{i}+\hat{j})$ (3) $\frac{2}{3}(\hat{i}+\hat{j})$ (4) $\frac{4}{3}(\hat{i}+\hat{j})$

23. Find the torque about the origin when a force of $3\hat{j}$ N acts on a particle whose position vector is $2\hat{k}$ m. **[NEET-2020]**

1) $6\hat{k}$ Nm 2) $6\hat{i}$ Nm 3) $6\hat{j}$ Nm 4) $-6\hat{i}$ Nm

24. A uniform rod of length 200 cm and mass 500 g is balanced on a wedge placed at 40 cm mark. A mass of 2 kg is suspended from the rod at 20 cm and another unknown mass 'm' is suspended from the rod at 160 cm mark as shown in the figure . Find the value of 'm' such that the rod is in equilibrium. $(g = 10\,m/s^2)$ **[NEET-2021]**



1. $\frac{1}{3}kg$ 2. $\frac{1}{6}kg$ 3. $\frac{1}{12}kg$ 4. $\frac{1}{2}kg$

25. From a circular ring of mass 'M' and radius 'R' an arc corresponding to a 90^0 sector is removed. The moment of inertial of the remaining part of the erring about an axis passing through the centre of the ring and perpendicular to the plane of the ring is 'K' times 'MR^2. Then the value of 'K' is [NEET-2021]

1) $\frac{7}{8}$ 2) $\frac{1}{4}$ 3) $\frac{1}{8}$ 4) $\frac{3}{4}$

26. Two objects of mass 10 kg and 20 kg respectively are connected to the two ends of a rigid rod of length 10 m with negligible mass. The distance of the center of mass of the system from the 10 kg mass is

1) $\frac{10}{3}m$ 2) $\frac{20}{3}m$ 3) 10 m 4) 5 m [NEET-2022]

27. The ratio of the radius of gyration of a thin uniform disc about an axis passing through its centre and normal to its plane to the radius of gyration of the disc about its diameter is [NEET-2022]

1) 2 : 1 2) $\sqrt{2}:1$ 3) 4 : 1 4) $1:\sqrt{2}$

28. A shell of mass m is at rest initially. It explodes into three fragments having mass in the ratio 2 : 2 : 1. If the fragments having equal mass fly off along mutually perpendicular directions with speed υ, the speed of the third (lighter) fragment is: [NEET-2022]

1) υ 2) $\sqrt{2}\upsilon$ 3) $2\sqrt{2}\upsilon$ 4) $3\sqrt{2}\upsilon$

NCERT LINE BY LINE QUESTIONS – ANSWERS

1. (3)	2. (2)	3. (4)	4. (4)	5. (3)	6. (2)	7. (2)	8. (4)	9. (1)	10. (3)
11. (4)	12. (2)	13. (1)	14. (1)	15. (1)	16. (3)	17. (3)	18. (2)	19. (3)	20. (3)

NCERT BASED PRACTICE QUESTIONS - ANSWERS

1) b	2) c	3) a	4) d	5) b	6) a	7) d	8) c	9) a	10) c
11) b	12) a	13) c	14) a	15) a	16) b	17) b	18) b	19) c	20) b
21) c	22) a	23) b	24) c	25) c	26) c	27) a	28) c	29) a	30) c
31) c	32) c	33) b	34) c	35) b	36) b	37) c	38) b	39) a	40) a

TOPIC WISE PRACTICE QUESTIONS - ANSWERS

1)	1	2)	1	3)	1	4)	2	5)	2	6)	3	7)	4	8)	3	9)	4	10)	2
11)	3	12)	1	13)	1	14)	4	15)	1	16)	1	17)	4	18)	2	19)	3	20)	2
21)	4	22)	1	23)	1	24)	4	25)	3	26)	2	27)	1	28)	2	29)	4	30)	2
31)	1	32)	3	33)	2	34)	4	35)	2	36)	4	37)	4	38)	3	39)	2	40)	4
41)	3	42)	2	43)	3	44)	4	45)	1	46)	2	47)	3	48)	1	49)	3	50)	3
51)	2	52)	3	53)	1	54)	3	55)	2	56)	1	57)	4	58)	2	59)	3	60)	4

NEET PREVIOUS YEARS QUESTIONS-ANSWERS

1)	1	**2)**	2	**3)**	2	**4)**	1	**5)**	4	**6)**	2	**7)**	1	**8)**	2	**9)**	2	**10)**	3
11)	3	**12)**	4	**13)**	3	**14)**	4	**15)**	4	**16)**	1	**17)**	1	**18)**	1	**19)**	4	**20)**	4
21)	1	**22)**	3	**23)**	4	**24)**	3	**25)**	4	**26)**	2	**27)**	2	**28)**	3				

TOPIC WISE PRACTICE QUESTIONS - SOLUTIONS

1. **(1)** Centre of mass of the system lies closer to the earth.

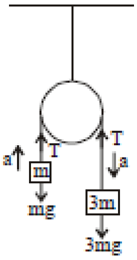
2. **(1)** The centre of mass of two particles lies always on the line joining the two particles.

3. **(1)** upward the radius

4. **(2)** From $v = r\omega$, linear velocities (v) for particles at different distances (r) from the axis of rotation are different.

5. **(2)** As net external force on the system is zero therefore position of their centre of mass remains unaffected i.e. they will hit each other at the point of centre of mass. The centre of mass of the system lies nearer to A because $M_A > M_B$.

6. **(3)** When the system is released, heavier mass move downward and the lighter one upward. Thus, centre of mass will move towards the heavier mass with acceleration



$$a = \left(\frac{3m - m}{3m + m}\right) g = \frac{g}{2}$$

7. **(4)** $\alpha = \dfrac{2\pi(n_2 - n_1)}{t} = \dfrac{2\pi\left(\dfrac{4500 - 1200}{60}\right)}{10} \text{rad}/s^2$

$$= \frac{2\pi \dfrac{3300}{60}}{10} \times \frac{360}{2\pi} \frac{\text{degree}}{s^2} \quad \alpha = 1980 \,\text{degree}/s^2$$

8. **(3)** We can assume that three particles of equal mass m are placed at the corners of triangle

$\vec{r}_1 = 0\hat{i} + 0\hat{j}, \vec{r}_2 = b\hat{i} + 0\hat{j}$ and $\vec{r}_3 = 0\hat{i} + h\hat{j}$

$$\therefore \overline{r_{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3} = \frac{b}{3}\hat{i} + \frac{h}{3}\hat{j}$$

i.e. coordinates of centre of mass is $\left(\dfrac{b}{3}, \dfrac{h}{3}\right)$

9. (4) <u>1st case:</u>

We can write the equation of motion for circular motion as:

$\omega^2 = \omega_0^2 + 2a\theta_n$

Now, $\omega = \dfrac{\omega_0}{2}, \theta = 36 \times 2\pi$ (given)

So, $\dfrac{\omega_0^2}{4} = \omega_0^2 + 2\alpha \times 36 \times 2\pi$

So, $\alpha = \dfrac{-3\omega_0^2}{4 \times 144\pi}$

<u>2nd case:</u>

$0 = \dfrac{\omega_0^2}{4} - 2 \times \dfrac{3\omega_0^2}{4 \times 144\pi} \times \theta$

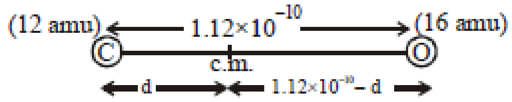
So, $\theta = 24\pi$

So, number of rotations made by the fan before coming to rest $= \dfrac{24\pi}{2\pi} = 12$

10. **(2)** $\vec{r} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = \dfrac{m(\vec{x} + \vec{y})}{2m} = \dfrac{\vec{x} + \vec{y}}{2}$

11. **(3)** The velocity of top point of the wheel is twice that of centre of mass. And the speed of centre of mass is same for both the wheels.

12. **(1)**

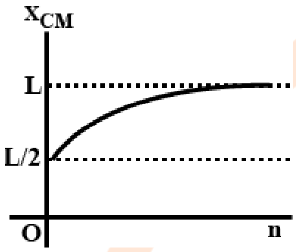


From definition of centre of mass

$$d=\frac{16\times1.12\times10^{-10}+12\times0}{16+12}=0.64\times10^{-10}\,m$$

13. **(1)** $p=K\left(\frac{x}{L}\right)^2$ for n=0, p= constant ⇒XCM=L/2

for $n\to\infty$, $X_{CM}\to L$



14. **(4)** Force F_A on particle A is given by

$F_A=m_Aa_A=\frac{m_Av}{t}$ -------------(1)

Similarly $F_B=m_Ba_B=\frac{m_B\times2v}{t}$ -------(2)

Now $\frac{m_Av}{t}=\frac{m_B\times2v}{t}$ $(\because F_A=F_B)$

So $m_A=2m_B$

For the centre of mass of the system

$$v=\frac{m_Av_A+m_Bv_B}{m_A+m_B}\text{ or }v=\frac{2m_Bv-m_B\times2v}{2m_B+m_B}=0$$

Negative sign is used because the particles are travelling in opposite directions.

15. **(1)** We may consider the entire mass of the stick to be concentrated as a point mass at the centre of mass of the stick. The centre of mass moves as a projectile, it will move along a parabolic path.

16. **(1)** σ = mass per unit area
Mass of complete 6cm radius
Disk = $\sigma(\pi(6)^2)$
Mass of small disk= $\sigma(\pi(2)^2)$.
Centers are O and C.

$$\text{Shift in com }==\frac{-\sigma\left(\pi(2)^2\right)\times3.2}{\sigma\left(\pi(6)^2\right)-\sigma\left(\pi(2)^2\right)}=\frac{4\pi\times3.2}{32\pi}=\frac{3.2}{8}=0.4cm$$

17. **(4)** Force in linear motion corresponds to torque in rotational motion.

18. **(2)** conserved about the point of contact only

19. **(3)** L = m v r or v = L /mr

Centripetal force $\frac{mv^2}{r}=\frac{m(L/mr)^2}{r}=\frac{L^2}{mr^3}$

20. **(2)** Conceptual

21. **(4)** As angular momentum, $\vec{L}=\vec{r}\times\vec{p}$, therefore, direction of $\vec{L}$ is along a line perpendicular to the plane of rotation.

22. **(1)** $L=mvr=m\omega r^2=m(2\pi f)r^2$

$$=0.2\times2\pi\times\frac{2}{\pi}\times(1)^2=0.2\times4=0.8\,kg-m^2/s$$

23. (1) As $\tau_{external}=0\Rightarrow L=Iw=$ constant

Now dropping the masses will decrease Moment of inertia (I) of the system thus increasing his angular velocity(w).

24. **(4)** $\tau \times \Delta t = L_0 \quad \{\because \text{since } L_f = 0\}$

$\Rightarrow \tau \times \Delta t = I\omega$

or $\tau \times 60 = 2 \times 2 \times 60\pi / 60 \left(\because f = 60\text{rpm} \quad \tau = \frac{\pi}{15} N - m \quad \therefore \omega = 2\pi f = 2\pi \times \frac{60}{60}\right)$

25. **(3)** Since the spheres are smooth, there will be no transfer of angular momentum from the sphere A to sphere B. The sphere A only transfers its linear velocity v to the sphere B and will continue to rotate with the same angular speed ω.

26. **(2)** $v = \sqrt{Kr}.t$

$L = mvr = m\sqrt{Kr^3}t \Rightarrow \frac{dL}{dt} = m\sqrt{Kr^3}$

27. **(1)** Angular momentum $L = m(v \times r)$

$= 2kg\left(\frac{dr}{dt} \times r\right) = 2kg\left(4t\hat{j} \times 5\hat{i} - 2t^2\hat{j}\right)$

$= 2kg\left(-20t\hat{k}\right) = 2kg \times -20 \times 2m^{-2}s^{-1}\hat{k} = -80\hat{k}$

28. (2) Angular momentum imparted by bullet on the door= mvr

$= \left(10 \times 10^{-3}\right) \times 500 \times 0.5\, kgm^2 / s$

Moment of inertia of the door,

$I = ML^2 / 3 = \frac{1}{3} \times 12 \times 1^2 = 4\, kgm^2$

Angular momentum of the system after the bullet gets embedded ≈ Iω

From conservation of angular momentum about the rotation axis,

mvr = Iω $\Rightarrow$ ω=0.625rad/s

29. **(4)** Angular momentum is constant

$\Rightarrow mr^2\omega = \text{const} \Rightarrow \omega = \frac{\text{const}}{mr^2}$

$T = m\omega^2 r = m\left(\frac{\text{const}}{mr^2}\right)^2 r = (\text{const})r^{-3}$

thus, $n = -3$

30. **(2)** α is proportional to ω

Let $\alpha = k\omega$ $\quad$ ($\because$ k is a constant)

$\frac{d\omega}{dt} = k\omega \qquad \left[\text{also } \frac{d\theta}{dt} = \omega \Rightarrow dt = \frac{d\theta}{\omega}\right]$

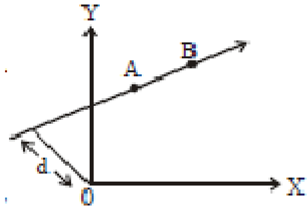
$\therefore \frac{\omega d\omega}{d\theta} = k\omega \Rightarrow d\omega = kd\theta$ Now $\int_{\omega}^{\omega/2} d\omega = k\int d\theta$

$\int_{\omega/2}^{0} d\omega = k\int_0^{\theta} d\theta \Rightarrow -\frac{\omega}{2} = k\theta \Rightarrow -\frac{\omega}{2} = k\theta_1 \; (\because \theta_1 = 2\pi n)$

$\therefore \theta = \theta_1$ or $2\pi n_1 = 2\pi n \Rightarrow n_1 = n$

31. **(1)** Angular momentum = linear momentum × distance of line of action of linear momentum about the origin.

$L_A = P_A \times d$ and $L_B = P_B \times d$



As linear momenta are equal, therefore, $L_A = L_B$.

32. **(3)** $\vec{\tau} = \frac{d\vec{J}}{dt} = \frac{d}{dt}\left(a\hat{i} + bt^2\hat{j}\right) = 2bt\hat{j}$

$$\cos 45° = \frac{\vec{\tau}.\vec{J}}{\tau J} \text{ or } \frac{1}{\sqrt{2}} = \frac{(2b+\hat{j}).(a\hat{i}+bt^2\hat{j})}{2bt \times \sqrt{a^2+b^2t^4}}$$

$$\text{or } t = \sqrt{\frac{a}{b}} \text{ and } \vec{\tau} = 2b \times \sqrt{\frac{a}{b}}\hat{j}.$$

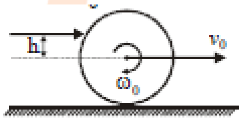
33. **(2)** A raw egg behaves like a spherical shell and a half boiled egg behaves like a solid sphere

$$\therefore \frac{I_r}{I_s} = \frac{2/3mr^2}{2/5mr^2} = \frac{5}{3} > 1$$

34. **(4)** When the ball is hit by a cue, the linear impulse imparted to the ball = change in momentum = mv_0

Angular momentum = Moment of momentum

$$I\omega_0 = (mv_0)h$$

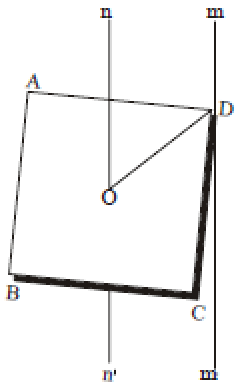


$$\frac{2}{5}mr^2\omega_0 = mv_0h \text{ or } \omega_0 = \frac{5v_0h}{2r^2}$$

35. **(2)** Use theorem of parallel axes.

36. **(4)** $I_{nn'} = \frac{1}{12}M\left(a^2+a^2\right) = \frac{Ma^2}{6}$

Also, $DO = \frac{DB}{2} = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$



According to parallel axis theorem

$$I_{mm'} = I_{nn'} + M\left(\frac{a}{\sqrt{2}}\right)^2$$

$$= \frac{Ma^2}{6} + \frac{Ma^2}{2} = \frac{2}{3}Ma^2$$

37. (4) The magnitude of angular momentum of a strong body is given by:

If no torque acts, the angular momentum is conserved, i.e., $I\omega$ =constant

$$\therefore I_1\omega_1 = I_2\omega_2$$

K_1 and K_2 are the corresponding radii of gyration, then

$I_1 = MK_1^2,\ I_2 = MK_2^2$

or, $I_1\omega_1 = I_2\omega_2$

or, $MK_1^2\omega_1 = MK_2^2\omega_2$

$$\frac{K_1}{K_2} = \frac{\sqrt{\omega_2}}{\sqrt{\omega_1}}$$

or $K_1 : K_2 = \sqrt{\omega_2} : \sqrt{\omega_1}$

38. **(3)** Since moment of inertia $I = \dfrac{mr^2}{2}$, where m is the mass of body & r is distance of it from the fixed axis.

mass = density × volume $\{\because \rho = \text{density}\}$

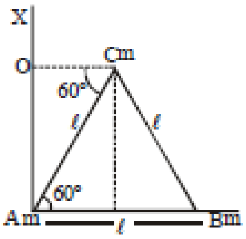
$M_A = \rho\pi r^2 t;\ I_A = \dfrac{\rho\pi r^4 t}{2}$

$M_B = \rho\pi r^2 \times 16 \times t/4 = 4\rho\pi r^2 t$

$I_B = \dfrac{M_B(4r)^2}{2}$

$= \dfrac{4\rho\pi r^2 t.16r^2}{2} = \dfrac{64\rho\pi r^4 t}{2}$

$\therefore I_B > I_A$



39. (2) The moment of inertia of the disc about an axis parallel to its plane is

$$I_t = I_d + MR^2 \Rightarrow I = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$$

or $MR^2 = \dfrac{4I}{5}$

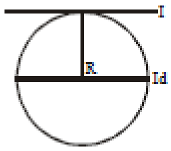
Now, moment of inertia about a tangent perpendicular to its plane is

$$I^| = \frac{3}{2}MR^2 = \frac{3}{2}\times\frac{4}{5}I = \frac{6}{5}I$$

40. **(4)** Melting of ice produces water which will spread over larger distance away from the axis of rotation. This increases the moment of inertia so angular velocity decreases

41. **(3)** Moment of inertia of disc about its diameter is

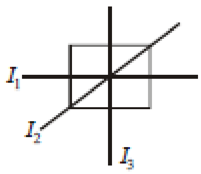
$$I_d = \frac{1}{4}MR^2$$



MI of disc about a tangent passing through rim and in the plane of disc is

$$I = I_G + MR^2 = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$$

42. **(4)** For a thin uniform square sheet



$$I_1 = I_2 = I_3 = \frac{ma^2}{12}$$

43. **(3)** M.I. of uniform circular disc about diameter = I, According to the theorem of perpendicular axes. M.I. of disc about its axis $2I\left(=\frac{1}{2}mr^2\right)$

Applying theorem of || axes

M.I. of disc about the given axis = 2 I + m r^2 = 2 I + 4I = 6 I

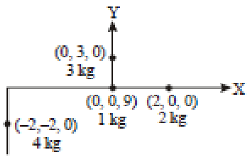
44. **(4)** $I=\frac{1}{2}mR^2 \quad M\propto t\propto R^2$

For disc X, $I_X=\frac{1}{2}(m)(R)^2=\frac{1}{2}(\pi r^2 t)(R)^2$

For disc Y, $I_Y=\frac{1}{2}\left[\pi(4R)^2.t/4\right][4R]^2$

$\Rightarrow \frac{I_X}{I_Y}=\frac{1}{(4)^3}\Rightarrow I_Y=64I_X$

45. **(1)** Moment of inertia of the whole system about the axis of rotation will be equal to the sum of the moments of inertia of all the particles.



$\therefore I = I_1 + I_2 + I_3 + I_4$

$= 0 + 0 + 27 + 16 = 43$ kg m^2

46. **(2)** MOI is $\sum m_i r_i^2$. About BC masses are spread far away than any other axis.

47. **(3)** $L_1=L_2$; $I_1\omega_1=I_2\omega_2$

$\frac{MR^2}{2}\omega_1=\left(\frac{MR^2}{2}+mR^2+mR^2\right)\omega_2$

$\frac{M}{2}\omega_1=\frac{(M+4m)}{2}\omega_2$

$\omega_2=\left(\frac{M}{M+4m}\right)\omega_1$

48. **(1)** For solid sphere $\frac{K^2}{R^2}=\frac{2}{5}$

For disc & solid cylinder $\frac{K^2}{R^2}=\frac{1}{2}$

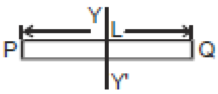
Since acceleration of a body, which is rolling on an inclined plane at angle q with horizontal is $a=\frac{g\sin\theta}{1+K^2/R^2}$ ....(i)

It is clear from eq.(i) that

$a_{\text{solid sphere}} > a_{\text{disc}} = a_{\text{solid cylinder}}$

hence solid sphere take least time in reaching the bottom of the inclined plane.

49. **(3)** If rotation axis is passing through its middle point & is $\perp$ to its plane, then moment of inertia about YY' is $I=\frac{ML^2}{12}$

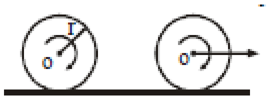


where M = volume × density = (L×A)× ρ

so $I=\frac{L^3A\rho}{12}$ so rotational K.E. $=\frac{1}{2}I\omega^2=\frac{L^3A\rho\omega^2}{24}$

50. **(3)** From conservation of angular momentum about any fix point on the surface,

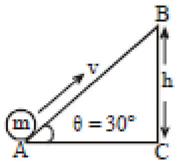
$$mr^2\omega_0 = 2mr^2\omega$$



$$\Rightarrow \omega = \omega_0/2 \Rightarrow v = \frac{\omega_0 r}{2} \qquad [\because v = r\omega]$$

51. **(2)** $$\frac{\text{Rotational K.E}}{\text{Total K.E}} = \frac{\frac{1}{2}mv^2\left(\frac{K^2}{R^2}\right)}{\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)} = \frac{K^2}{K^2+R^2}$$

52. **(3)** If a body rolls on a horizontal surface, it possesses both translational and rotational kinetic energies. The net kinetic energy is given by



$$K_{net} = \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$$ where K is the radius of gyration.

So from law of conservation of energy

$$\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right) = mgh$$

where h is the height attained by the sphere.

i.e., $$\frac{1}{2}\times 2\times(10)^2\left(1+\frac{2}{5}\right) = 2\times 9.8\times h$$

or $$h = \frac{700}{98} = 7.1\text{m}$$

53. **(1)** Velocity of a body rolling down an inclined plane is given by

$$a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$$

In case of a solid sphere, we have

$$\frac{K^2}{R^2} = \frac{[(I/M)]}{R^2} = \frac{I}{MR^2} = \frac{(2/5)MR^2}{MR^2} = \frac{2}{5}$$

Substituting $\frac{K^2}{R^2} = \frac{2}{5}$, we get

$$a = \frac{5}{7}g\sin\theta$$

54. **(3)** $$\mu = \frac{F}{R} = \frac{mg\sin\alpha}{mg\cos\alpha} = \tan\alpha$$

55. **(2)** $$v_R = \sqrt{v^2+v^2+2v^2\cos\theta}$$

$$= \sqrt{2v^2(1+\cos\theta)} = 2v\cos\frac{\theta}{2}$$

56. **(1)** $$E_r = \frac{1}{2}I\omega^2 = \frac{1}{2}\times 3\times(3)^3 = 13.5\text{J}$$

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}\times 27\times v^2 = 13.5 \Rightarrow v = 1\text{m/s}$$

57. **(4)** Minimum velocity for a body rolling without slipping

$$v = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}$$

For solid sphere $\frac{K^2}{R^2} = \frac{2}{5}$

$$\therefore v = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}} = \sqrt{\frac{10}{7}gh}$$

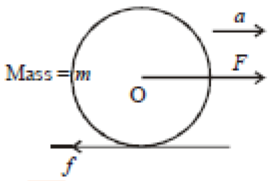
58. **(2)** $I = 1.2 \text{ kg m}^2$, $E_r = 1500$ J,
$\alpha = 25$ rad /sec^2, $\omega_1 = 0$, t =?

As $E_r = \frac{1}{2}I\omega^2$

$$\omega = \sqrt{\frac{2E_r}{I}} = \sqrt{\frac{2\times1500}{1.2}} = 50\,\text{rad/sec}$$

From $\omega = \omega_1 + \alpha t$, $50 = 0 + 25t, \Rightarrow t = 2$ sec

59. **(3)** From figure,



$ma = F - f$ ---------------(i)

And, torque $\tau = I\alpha$

$$\frac{mR^2}{2}\alpha = fR$$

$$\frac{mR^2}{2}\frac{a}{R} = fR\left[\because \alpha = \frac{a}{R}\right]$$

$\frac{ma}{2} = f$ -----------(ii)

Put this value in equation (i),

$$ma = F - \frac{ma}{2} \text{ or } F = \frac{3ma}{2}$$

60. **(4)** $\frac{\text{K.E. of rotation}}{\text{K.E. of translation}} = \frac{40}{100} = \frac{2}{5}$

i.e., $\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2} = \frac{1/2I\omega^2}{1/2mr^2\omega^2} = \frac{2}{5}$ or $I = \frac{2}{5}mr^2$

Hence the body is a solid sphere.

NEET PREVIOUS YEARS QUESTIONS-EXPLANATIONS

1. **(1)** Work done required to bring them rest

$\Delta W = \Delta KE$ (work-energy theorem)

$$\Delta W = \frac{1}{2}I\omega^2 \left(\Delta kE_{rot} = \frac{1}{2}I\omega^2\right)$$

or $\Delta W \propto I$ (for same ω)

$$I_{solid\ sphere} = \frac{2}{2}MR^2$$

$$I_{Disk} = \frac{1}{2}MR^2; I_{Ring} = MR^2$$

$$\therefore W_C > W_B > W_A$$

2. **(2)** In rolling motion, rotational kinetic energy.

$$K_t = \frac{1}{2}mv^2$$

and $K_t + K_r = \frac{1}{2}mv^2 + \frac{1}{2}l\omega^2$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{7}{10}mv^2$$

$$\therefore \frac{K_t}{K_t + K_r} = \frac{\frac{1}{2}mv^2}{\frac{7}{10}mv^2} = \frac{5}{7}$$

3. **(2)** $I = MR^2$; $\tau = FR$; $\tau = l\alpha$

$$FR = MR^2 d$$

$$\alpha = \frac{F}{MR} = \frac{30}{3 \times 0.4} = 25\,rad/s^2$$

4. **(1)** Here, $I\omega_1 + I\omega_2 = 2I\omega$

$$\Rightarrow \omega = \frac{\omega_1 + \omega_2}{2}$$

$$(K.E.)_i = \frac{1}{2}I\omega_1^2 + \frac{1}{2}I\omega_2^2$$

$$(K.E.)_f = \frac{1}{2}2I\omega^2 = I\left(\frac{\omega_1 + \omega_2}{2}\right)^2$$

Loss in K.E. = $(K.E)_f - (K.E)_i = \frac{1}{4}I(\omega_1 - \omega_2)^2$

5. **(4)** Centre of mass may or may not coincide with centre of gravity. Net torque of gravitational pull is zero about centre of mass $\tau_g = \sum \tau_i = \sum r_i \times m_{ig} = 0$

Mechanical advantage , $M.A = \frac{Load}{Effort}$

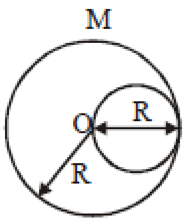
If M.A. > 1 $\Rightarrow$ Load > Effort

6. **(2)** Moment of inertia of complete disc about point 'O'.

$$I_{Total\ disc} = \frac{MR^2}{2}$$

Mass of removed disc

$$M_{Removed} = \frac{M}{4} \text{ (Mass} \propto \text{area)}$$



Moment of inertia of removed disc about point 'O'.

$I_{Removed}$ (about same perpendicular axis) = $I_{cm} + mx^2$

$$= \frac{M}{4}\frac{(R/2)^2}{2} + \frac{M}{4}\left(\frac{R}{2}\right)^2 = \frac{3MR^2}{32}$$

Therefore the moment of inertia of the remaining part of the disc about a perpendicular axis passing through the centre,

$I_{Remaing\ disc} = I_{Total} - I_{Removed}$

$$= \frac{MR^2}{2} - \frac{3}{32}MR^2 = \frac{13}{32}MR^2$$

7. (1) Given: Radius of disc, R = 50 cm

angular acceleration a = 2.0 rad s^{-2}; time t = 2s Particle at periphery (assume) will have both radial (one) and tangential acceleration

at = Rα = 0.5 × 2 = 1 m/s^2

From equation,

$$\omega = \omega_0 + \alpha t$$

$$\omega = 0 + 2 \times 2 = 4 \text{ rad/sec}$$

$a_c = \omega^2 R = (4)^2 \times 0.5 = 16 \times 0.5 = 8 m/s^2$

Net acceleration,

$$a_{total} = \sqrt{a_t^2 + a_c^2} = \sqrt{1^2 + 8^2} \approx 8 m/s^2$$

8. **(2)** Time of descent $\propto \frac{K^2}{R^2}$

Order of value of $\frac{K^2}{R^2}$

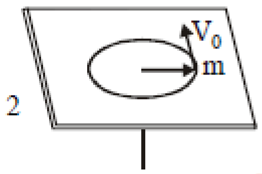
for disc; $\frac{K^2}{R^2} = \frac{1}{2} = 0.5$

for sphere; $\frac{K^2}{R^2} = \frac{2}{5} = 0.4$

(sphere) < (disc)

∴ Sphere reaches first

9. **(2)** Applying angular momentum conservation



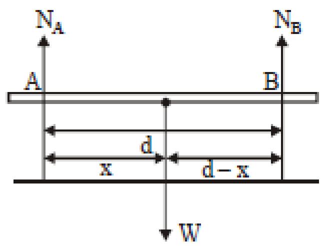
$$mV_0R_0 = (m)(V^1)\left(\frac{R_0}{2}\right); \therefore v^{|} = 2V_0$$

Therefore, new KE $= \frac{1}{2}m(2V_0)^2 = 2mv_0^2$

10. **(3)** By torque balancing about B

$$N_A(d) = W(d - x)$$

$$N_A = \frac{W(d-x)}{d}$$



11. **(3)** Moment of inertia of shell 1 along diameter

$I_{diameter} = \frac{2}{3}MR^2$

Moment of inertia of shell 2 = m. i of shell 3

$= I_{tangential} = \frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2$

So, I of the system along x x^1

= $I_{diameter}$ + ($I_{tangential}$) × 2

or $I_{total} = \frac{2}{3}MR^2 + \left(\frac{5}{3}MR^2\right) \times 2$

$= \frac{12}{3}MR^2 = 4MR^2$

12. **(4) Given :** Speed V = 54 kmh^{-1} = 15 ms^{-1}

Moment of inertia, I = 3 kgm^2

Time t = 15s

$\omega_i = \frac{V}{r} = \frac{15}{0.45} = \frac{100}{3}$, $\omega_f = 0$

$\omega_f = \omega_i + \alpha t$

$0 = \frac{100}{3} + (-\alpha)(15) \Rightarrow \alpha = \frac{100}{45}$

Average torque transmitted by brakes to the wheel

$\tau = (I)(\alpha) = 3 \times \frac{100}{45} = 6.66\, kgm^2 s^{-2}$

13. **(3)** Work required to set the rod rotating with angular velocity ω_0

$K.E = \frac{1}{2} I\omega^2$

Work is minimum when I is minimum.

I is minimum about the centre of mass

So, $(m_1)(x) = (m_2)(L - x)$

or, $m_1x = m_2L - m_2x$

$\therefore x = \frac{m_2 L}{m_1 + m_2}$

14. (4) From Newton's second law for rotational motion,

$\vec{\tau} = \frac{d\vec{L}}{dt}$, if $\vec{L} = $ constant then $\vec{\tau} = 0$

So, $\vec{\tau} = \vec{r} \times \vec{F} = 0$

$(2\hat{i} - 6\hat{j} - 12\hat{k}) \times (\alpha\hat{i} + 3\hat{j} + 6\hat{k}) = 0$

Solving we get $\alpha = -1$

15. **(4)** Here a = 2 revolutions/s^2 = 4π rad/s^2 (given)

$I_{cylinder} = \frac{1}{2}MR^2 = \frac{1}{2}(50)(0.5)^2 = \frac{25}{4} Kg - m^2$

As $\tau = I\alpha$ so $TR = I\alpha$

$\Rightarrow T = \frac{I\alpha}{R} = \frac{\left(\frac{25}{4}\right)(4\pi)}{(0.5)} N = 50\pi N = 157n$

16. **(1)** For solid sphere rolling without slipping on inclined plane, acceleration

$$a_1 = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$$

For solid sphere slipping on inclined plane without rolling, acceleration $a_2 = g\sin\theta$

Therefore required ratio $= \frac{a_1}{a_2}$

$$= \frac{1}{1+\frac{K^2}{R^2}} = \frac{1}{1+\frac{2}{5}} = \frac{5}{7}$$

17. $W_{all} = DKE$

$$\Rightarrow W = 0 - \frac{1}{2}mv_{cm}^2\left(1+\frac{K^2}{R^2}\right)$$

$$\Rightarrow W = -3J$$

18. $\theta = 2\pi \times 2\pi$ radian

$$\omega_0 = 3 \text{ rpm} \Rightarrow \frac{2\pi}{60}(3)\frac{\text{rad}}{\text{sec}}$$

$$\omega^2 = \omega_0^2 - 2\alpha\theta$$

$$0 = \left(\frac{3\times 2\pi}{60}\right)^2 - 2\alpha(4\pi^2)$$

$$\therefore \quad \alpha = \frac{1}{800}\text{ rad/s}^2$$

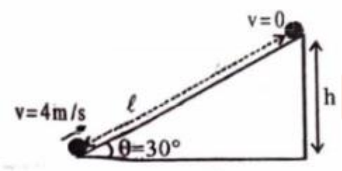
$$\tau = \frac{mR^2}{2}\alpha = \frac{2}{2}\times\left(\frac{4}{100}\right)^2\times\frac{1}{800} = 2\times 10^{-6}\text{ Nm}$$

19. $T_A = T_B$

$$\Rightarrow \frac{2\pi}{\omega_A} = \frac{2\pi}{\omega_B}$$

$$\Rightarrow \frac{\omega_A}{\omega_B} = 1 : 1$$

20.



$$\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right) = mgh$$

$$\Rightarrow 8\left(1+\frac{1}{2}\right) = 10h$$

$$h = 1.2m$$

$$\frac{h}{l} = \sin 30^0;\ l = 2.4cm$$

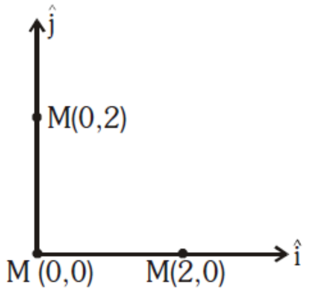
21. $$\omega_0 = \frac{360}{60}\text{rps} = 12\pi\,\text{rad s}^{-1}$$

$$\omega = \frac{1200}{60}\text{rps} = 40\pi\,\text{rad s}^{-1}$$

By using $\omega = \omega_0 + \alpha t$ we have $\alpha = \frac{\omega - \omega_0}{t}$

$\Rightarrow \alpha = \frac{28\pi}{14} = 2\pi \, rad \, s^{-2}$

22.



$X_{CM} = \frac{M \times 0 + M \times 2 + M \times 0}{3M} = \frac{2}{3}$

$Y_{CM} = \frac{M \times 0 + M \times 2 + M \times 0}{3M} = \frac{2}{3}$

Position vector $\vec{r} = X_{CM}\hat{i} + Y_{CM}\hat{j} = \frac{2}{3}\hat{i} + \frac{2}{3}\hat{j}$

23. $\vec{F} = 3\hat{j}N$, $\vec{r} = 3\hat{k}$

$\vec{\tau} = \vec{r} \times \vec{F} = 2\hat{k} \times 3\hat{j} = 6(\hat{k} \times \hat{j})$

$= 6(-\hat{i})$

$\vec{\tau} = -6\hat{i}N - m$

24.

$2 \times 20 = 0.5 \times 60 + mx120$

$40 - 30 = mx120$

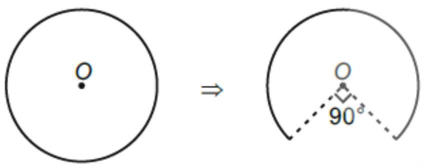
$10 = mx120$

$m = \frac{1}{12}$

Given that,

25.

Mass of Ring = M; Radius of Ring = R
Now 90° arc is removed from circular ring, then mass removed =M/4



Mass of remaining portion = $\frac{3M}{4}$

Moment of inertia of remaining part $= \int dmr^2$

$\Rightarrow I = R^2 \int dm \qquad (\because r = R)$

$\Rightarrow I = \frac{3MR^2}{4}$. So the value of K is $\frac{3}{4}$

26 $\frac{m_1}{m_2} = \frac{d_2}{d_1} = \frac{10 - d_1}{d_1}$

$\frac{10}{20} = \frac{10 - d_1}{d_1}$

$10d_1 = 200 - 20d_1$

$d_1 = \frac{10}{3}$

27. Co $MK_1^2 = \frac{1}{2}MR^2$

$MK_2^2 = \frac{1}{4}MR^2$

$\frac{K_1}{K_2} = \frac{1}{\sqrt{2}}$

28.: 1 $\bar{P}_3 = -\bar{P}_1 + \left(-\bar{P}_2\right)$

$m\bar{V}_1 = -2mV\hat{i} - 2mV\hat{j}$

$\Rightarrow \left|\bar{V}_1\right| = 2\sqrt{2}V$